

ARYAN SCHOOL OF ENGINEERING & ECHNOLOGY

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LECTURE NOTE

SUBJECT NAME- GEOTECHNICAL ENGINEERING

BRANCH-CIVIL ENGG.

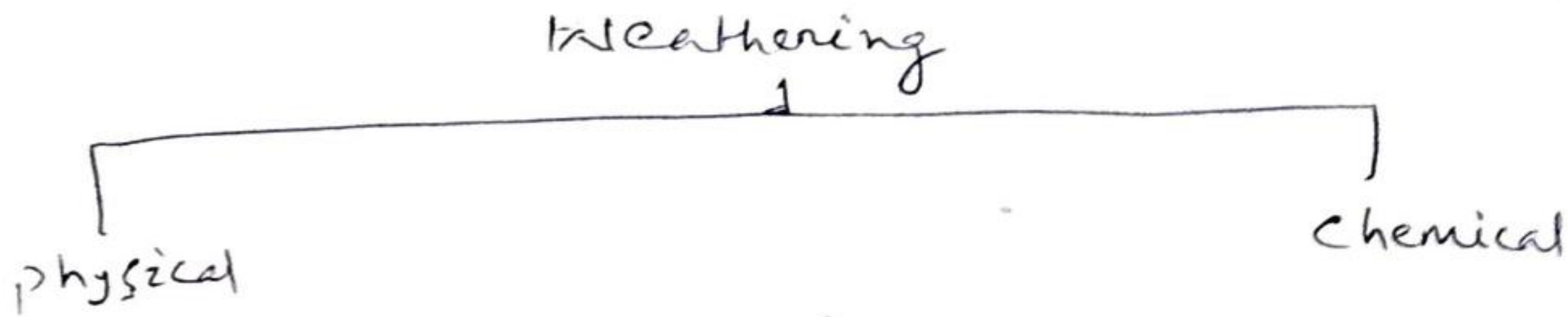
SEMESTER-3RD SEM

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Soil According to IS soil is any complex material produced by weathering of rock.

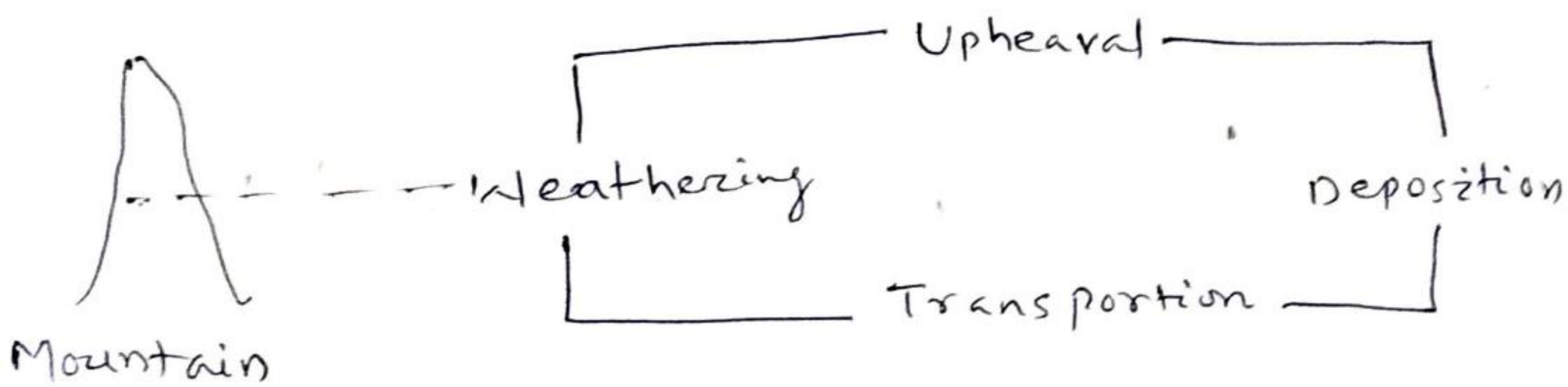
Weathering - Any changes that takes place in rock is called weathering.



physical In physical weathering it takes place due to temperature, pressure, air and water.

chemical In chemical weathering it takes place due to oxidation, reduction where chemical reaction takes place.

Steps for formation of soil



First of all near mountain, weathering of rock takes place. Then it should be transported and after that deposition of rock is formed. After deposition of rock upheaval process takes place.

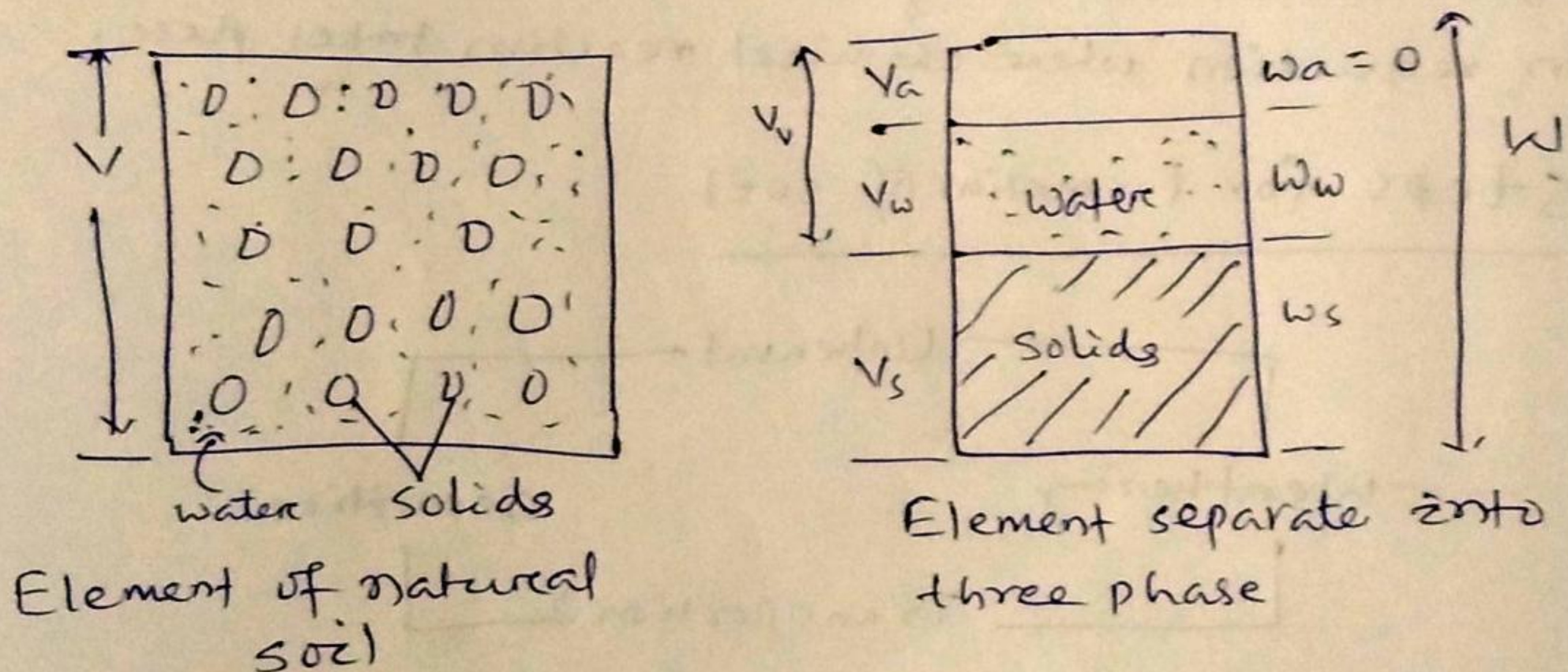
Upheaval is the disintegration of rock or a sudden changes in rock.

Foundation

Any component of structure which transfer load of structure to the soil. It is directly on the surface of soil. Any kind of structure is highway, building etc.

Definations and relationships

A soil mass is a three phase system consisting of solid particles, water and air. The void space between the soil grains is filled partly with water and partly with air.



Now V_a = volume of air

V_w = volume of water

V_s = volume of solids

Volume of void $V_v = V_a + V_w$

Hence total volume $V = V_s + V_w + V_a$

Similarly W_s = weight of solid

W_w = weight of water

W_a = weight of air = 0

Then total weight $W = W_s + W_w$

Water content (w)

It is defined as the ^{ratio of} weight of water to the weight of solids.

Mathematically $w = \frac{w_w}{w_s}$

$$\Rightarrow w = \frac{W - W_s}{w_s} = \frac{W}{w_s} - \frac{W_s}{w_s}$$

$$w = \frac{W}{w_s} - 1$$

Void ratio (e)

It is defined as the ratio of volume of voids to the volume of solids.

Mathematically $e = \frac{V_v}{V_s}$

$$\Rightarrow e = \frac{(V - V_s)}{V_s} = \frac{V - V_s}{V_s}$$

$$\Rightarrow e = \frac{V}{V_s} - \frac{V_s}{V_s}$$

$$\Rightarrow e = \frac{V}{V_s} - 1$$

Porosity (n)

The porosity 'n' of a soil sample is the ratio of volume of voids to the total volume of given soil mass.

Mathematically $n = \frac{V_v}{V}$

$$\Rightarrow n = \frac{V - V_s}{V} = \frac{V}{V} - \frac{V_s}{V}$$

$$\Rightarrow n = 1 - \frac{V_s}{V}$$

Density of soil The density of soil is defined as the mass of soil per unit volume.

Bulk density (ρ)

The bulk density is the total mass 'M' of the soil per unit of its total volume.

$$\text{Thus } \rho = \frac{M}{V}$$

It is expressed as g/cm^3 or kg/m^3

Dry density (ρ_d)

It is defined as the ratio of mass of solids per unit of total volume of soil mass.

$$\rho_d = \frac{M_s}{V}$$

Density of solids (ρ_s)

It is the ratio of mass of soil solids per unit volume of solids.

$$\rho_s = \frac{M_s}{V_s}$$

Specific gravity (G)

It is the ratio of unit weight of soil solids to that of water.

$$G = \frac{\gamma_s}{\gamma_w}$$

where γ_s = unit weight of soil solids

γ_w = unit weight of water

Relation between porosity (n) and void ratio (e)

$$n = \frac{V_v}{V}$$

Dividing throughout by V_s we get

$$\begin{aligned} n &= \frac{\frac{V_v}{V_s}}{\frac{V}{V_s}} = \frac{e}{\frac{V_s + V_v}{V_s}} \\ &= \frac{e}{\frac{V_s}{V_s} + \frac{V_v}{V_s}} = \frac{e}{1 + e} \end{aligned}$$

$$\Rightarrow \boxed{n = \frac{e}{1 + e}}$$

Similarly $e = \frac{V_v}{V_s}$

Dividing throughout by 'v' we get

$$e = \frac{\frac{V_v}{V}}{\frac{V_s}{V}} = \frac{n}{\frac{V - V_v}{V}}$$

$$\Rightarrow e = \frac{n}{\frac{V}{V} - \frac{V_v}{V}} = \frac{n}{1 - \frac{V_v}{V}}$$

$$\Rightarrow \boxed{e = \frac{n}{1 - n}}$$

Degree of saturation (S)

It is defined as the ratio of volume of water present in a given soil mass to the total volume of voids in it.

Mathematically $s = \frac{V_w}{V_v}$

The degree of saturation is usually expressed as a percentage and it is known as percentage saturation.

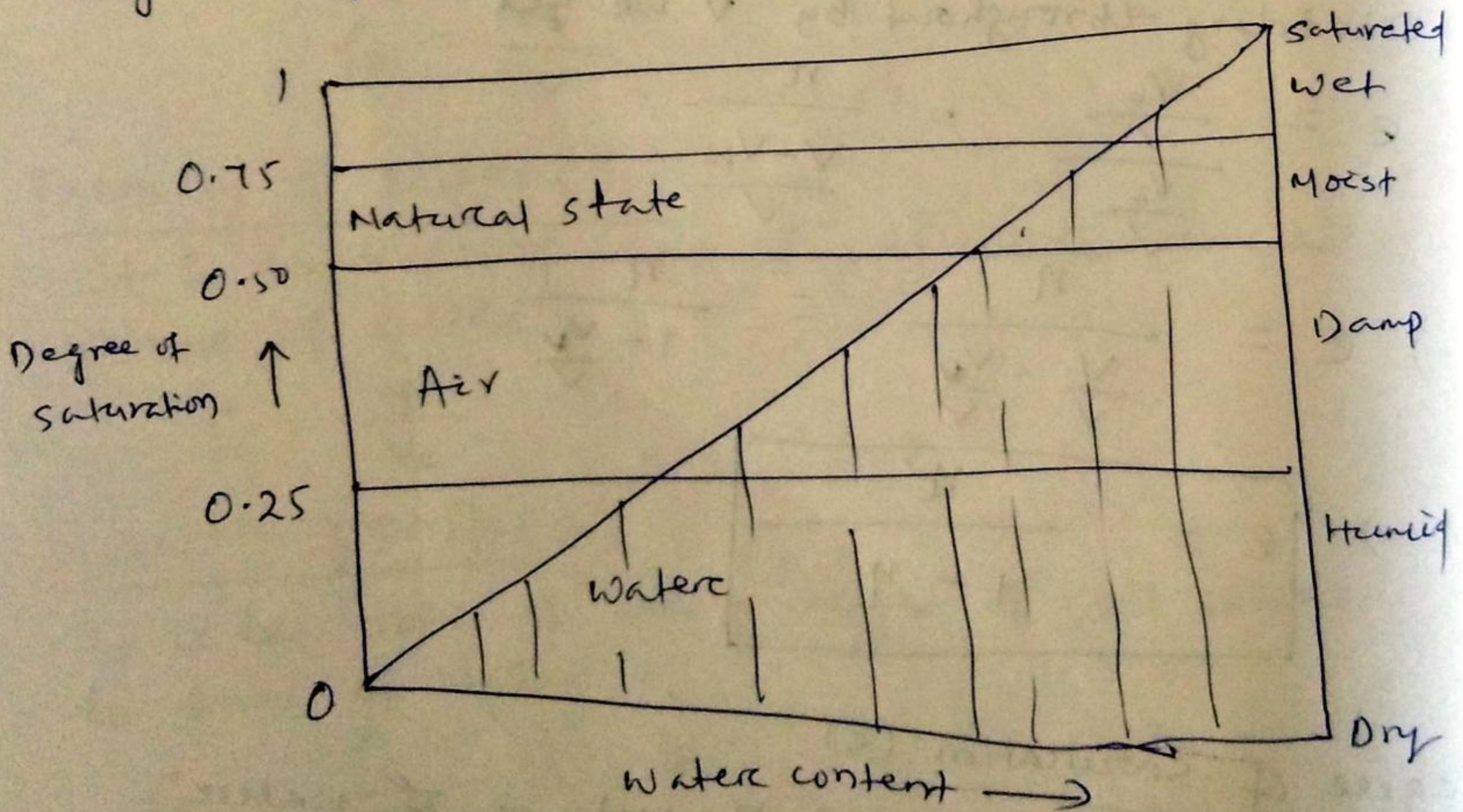
For a fully saturated sample $V_w = V_v$

Hence $s = \frac{V_w}{V_v} = \frac{V_v}{V_v} = 1$

For a perfectly dry sample $V_w = 0$

Hence $s = 0$

Depending upon degree of saturation a soil is generally described as dry, damp, moist, saturated etc.



percentage of air voids (n_a)

It is defined as the ratio of volume of air voids to the total volume of soil mass and it is expressed as percentage.

$$\text{Mathematically } n_a = \frac{V_a}{V} \times 100$$

Air content (a_e)

It is defined as the ratio of volume of air voids to the volume of voids.

$$a_e = \frac{V_a}{V_v} \quad \text{since } V_a = V_v - V_w$$

$$a_e = \frac{V_v - V_w}{V_v} = \frac{V_v}{V_v} - \frac{V_w}{V_v}$$

$$\Rightarrow a_e = 1 - \frac{V_w}{V_v}$$

$$\Rightarrow \boxed{a_e = 1 - S}$$

Density index and relative compaction

Density index is denoted as (I_D)

It is defined as the ratio of the difference between the voids ratio of the soil in its loosest state e_{max} and its natural voids ratio 'e' to the difference between the void ratios in the loosest and densest state.

$$\text{Mathematically } I_D = \frac{e_{max} - e}{e_{max} - e_{min}}$$

Where e_{max} = voids ratio in the loosest state

e_{min} = void ratio in densest state

e = natural void ratio of the deposit

The term I_D is used for cohesionless soil only.

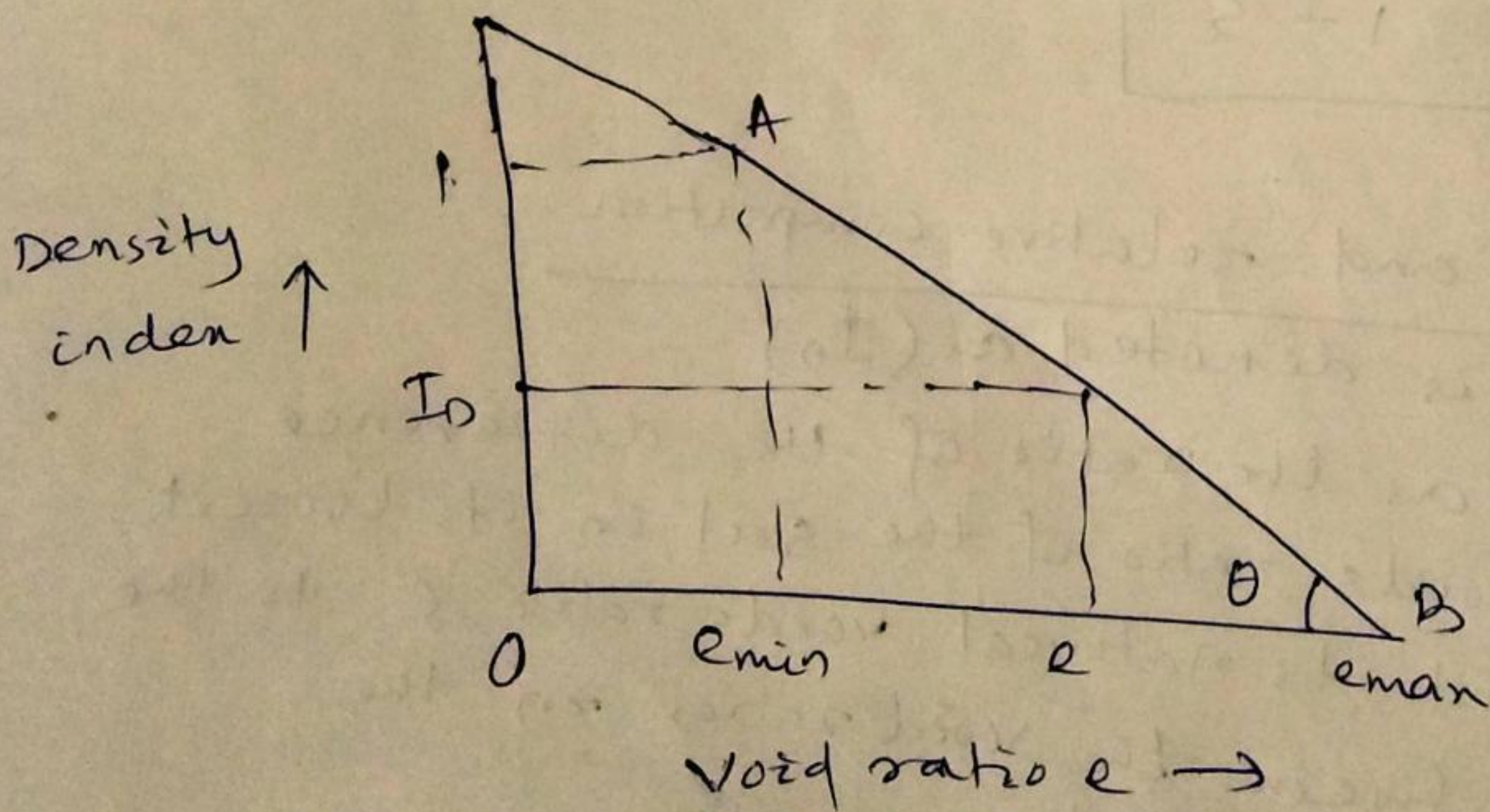
When the natural state of the cohesionless soil is in its loosest form then $e = e_{max}$

and hence $I_D = 0$

When the natural state of soil deposit is in its densest state $e = e_{min}$ and hence $I_D = 1$

For any intermediate state the density index varies between 0 and 1.

This relationship between I_D and e may be represented graphically.



The slope of the saturated straight line AB representing the relationship between I_D and e .

$$\tan \theta = \frac{1}{e_{\max} - e_{\min}}$$

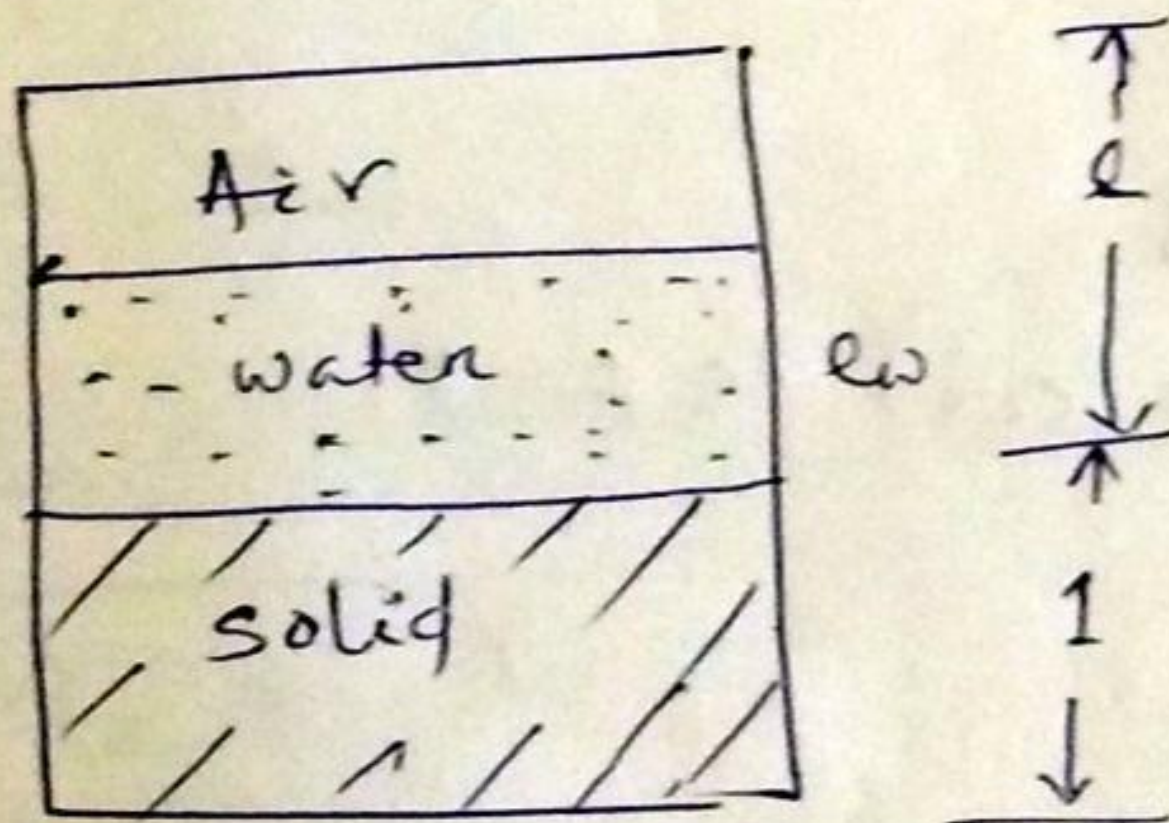
$$\Rightarrow e_{\max} - e_{\min} = \frac{1}{\tan \theta} = \cot \theta$$

$$I_D = \frac{e_{\max} - e}{e_{\max} - e_{\min}} = \frac{e_{\max} - e}{\cot \theta}$$

$$\Rightarrow \boxed{e_{\max} - e = I_D \cot \theta}$$

Functional relationship

Relationship between e , G , w and s



In the above figure ' lw ' represents the volume of water, ' e ' represents the volume of voids and volume of solids is equal to unity.

$$\text{Now } s = \frac{V_w}{V_v} = \frac{lw}{e}$$

$$\Rightarrow \boxed{lw = es} \quad \text{--- (1)}$$

The term lw is known as water voids ratio. For a fully saturated soil sample $lw = e$

$$\text{Then } w = \frac{w_w}{w_d} = \frac{w_w}{w_s}$$

$$= \frac{e w \gamma_w}{\gamma_s \cdot 1}$$

$$\text{But } G = \frac{\gamma_s}{\gamma_w} \Rightarrow \gamma_s = G \gamma_w \quad \text{--- (2)}$$

$$\text{Then } w = \frac{e w \gamma_w}{G \gamma_w}$$

$$\Rightarrow w = \frac{e w}{G}$$

$$\Rightarrow \boxed{e w = G w} \quad \text{--- (3)}$$

Equating equation (1) and (3) we get

$$G w = e s$$

$$\Rightarrow \boxed{e = \frac{w G}{s}}$$

For a fully saturated soil $s = 1$

$$\text{and } w = w_{\text{sat}}$$

$$\boxed{e = w_{\text{sat}} G}$$

Relation between e , s and n_a

We know that $n_a = \frac{V_a}{V}$

$$\text{But } V_a = V_v - V_w = e - ew$$

$$\text{and } V = V_s + V_v = 1 + e$$

$$\text{Then } n_a = \frac{e - ew}{1 + e}$$

$$\text{But } ew = es$$

$$n_a = \frac{e - es}{1 + e} \quad \Rightarrow \quad n_a = \frac{e(1 - s)}{1 + e}$$

Relation between n_a , a_c and n

We know that $a_c = \frac{V_a}{V_v}$

$$n = \frac{V_v}{V}$$

$$n_a = \frac{V_a}{V}$$

$$\Rightarrow n_a = \frac{V_a}{V_v} \times \frac{V_v}{V}$$

$$\Rightarrow \boxed{n_a = a_c \times n}$$

Relation between r_d , G and e

$$r_d = \frac{W_d}{V} = \frac{r_s V_s}{V}$$

But $V_s = 1$ and $V = (1 + e)$

$$r_d = \frac{r_s \cdot 1}{1 + e}$$

But $r_s = G r_w$

$$\boxed{r_d = \frac{G r_w}{1 + e}}$$

Q.1 A soil sample has a porosity of 40%. The specific gravity of solids is 2.70. Calculate void ratio, dry density. Also calculate unit weight of soil if it is 50% saturated and completely saturated?

Ans $n = 40\% = 0.4$, $G = 2.70$

$$e = \frac{n}{1-n} = \frac{0.4}{1-0.4} = 0.667$$

$$\text{Dry density } \gamma_d = \frac{G \gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+0.667}$$

$$= 15.89 \text{ kN/m}^3$$

$$e = \frac{G w}{s}$$

$$\Rightarrow w = \frac{e s}{G} = \frac{0.667 \times 0.5}{2.7} = 0.124$$

$$\gamma = \gamma_d (1+w) = 15.89 (1+0.124) = 17.85 \text{ kN/m}^3$$

$$w_{\text{sat}} = \frac{e}{G} = \frac{0.667}{2.7} = 0.247$$

$$\gamma_{\text{sat}} = \gamma_d (1+w_{\text{sat}})$$

$$= 15.89 (1+0.247)$$

$$= 19.81 \text{ kN/m}^3$$

Relationship

$$1) e = \frac{wG}{S}$$

e = void ratio G = specific gravity
 w = water content S = saturation

$$2) n_a = \frac{e(1-S)}{(1+e)}$$

$$3) n_a = n_{ae}$$

$$4) \gamma_d = \frac{G \gamma_w}{(1+e)}$$

$$5) \gamma_{sat} = \frac{(G+e) \gamma_w}{(1+e)}$$

$$6) \gamma_d = \frac{G \gamma_w}{(1+e)}$$

$$7) \gamma' = \gamma_{sat} - \gamma_w = \frac{(G-1) \gamma_w}{(1+e)}$$

$$8) \gamma_d = \frac{\gamma}{1+w}$$

$$9) \gamma' = \gamma_d - (1-n) \gamma_w$$

Determination of Index properties

The properties of soil which are used in their identification and classification. These include the determination of

- i) water content
- ii) specific gravity
- iii) Particle size distribution
- iv) Consistency limits
- v) Density index

These properties are known as index properties.

Water content determination

The water content of the soil is an important parameter. It is a quantitative measure of the wetness of the soil mass. The water content can be determined by any one of the following methods.

- 1) Oven drying method
- 2) Pycnometer method
- 3) Calcium carbide method
- 4) Sand bath method

Specific gravity determination

The specific gravity of solid particles is determined in the laboratory using following methods.

- 1) Density bottle method
- 2) Pycnometer method
- 3) Gas jar method
- 4) Measuring flask method

Measurement of mass density

The following methods are generally used for the determination of mass density.

- 1) Water displacement method
- 2) Core cutter method
- 3) Submerged mass density method
- 4) Sand replacement method

Particle size analysis

The mechanical analysis also known as particle size analysis is a method of separation of soils into different fractions based on particle size.

The mechanical analysis is done in two stages.

- 1) Sieve analysis
- 2) Sedimentation analysis

A soil sample may be either well graded or poorly graded.

A soil is said to be well graded when it has good representations of particles of all sizes.

A soil is said to be poorly graded if it has excess of certain particles and deficiency of other.

For a coarse grained soil certain particle size such as D_{10} , D_{30} and D_{60} are important.

D_{10} - It represents a size in mm such that 10% of the particles are finer than this size.

D_{60} - It represents a size in mm such that 60% of the particles are finer than this size.

D_{30} - It represents a size in mm such that 30% of the particles are finer than this size.

Uniformity Coefficient (C_u)

It is a measure of particle size range and is given by the ratio of D_{60} and D_{10} sizes.

Mathematically, Uniformity coefficient (C_u) = $\frac{D_{60}}{D_{10}}$

Coefficient of curvature (C_c)

$$C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}}$$

i) For a uniformly graded soil (C_u) is nearly unity.

ii) For a well graded soil (C_c) must be between 1 to 3.

iii) In case of gravels (C_u) must be greater than 4.

iv) Similarly for sand (C_u) should be greater than 6.

Gravel = 80mm to 4.75mm } Coarse grained

Sand = 4.75mm to 0.075mm }

Silt = 0.075mm to 0.002mm } Fine grained

Clay = < 0.002mm }

Q.1 A sieve analysis test was conducted in the laboratory from particle size distribution curve from following observations recorded. Calculate coefficient of curvature and coefficient of uniformity for the following data?

$$D_{10} = 0.32 \text{ mm} \quad D_{60} = 1.98$$
$$D_{30} = 1.25 \text{ mm}$$

Ans Coefficient of uniformity

$$C_u = \frac{D_{60}}{D_{10}} = \frac{1.98}{0.32}$$

$$\Rightarrow C_u = 6.187$$

Coefficient of curvature.

$$C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}}$$

$$= \frac{(1.25)^2}{1.98 \times 0.32}$$

$$= 2.46$$

Since $C_c > 1$, it is well graded.

Q.2 In the laboratory test, calculate coefficient of uniformity from following data?

$$D_{10} = 0.580 \text{ mm} \quad D_{60} = 0.60 \text{ mm}$$

Ans $C_u = \frac{D_{60}}{D_{10}} = \frac{0.60}{0.580}$

$$= 1.03 \approx 1$$

Hence it is uniformly graded.

plasticity characteristics of the soil

plasticity of soil

The plasticity of the soil is its ability to undergo deformation without cracking or fracturing. Plasticity of the soil is due to the presence of clay minerals.

Consistency of soils

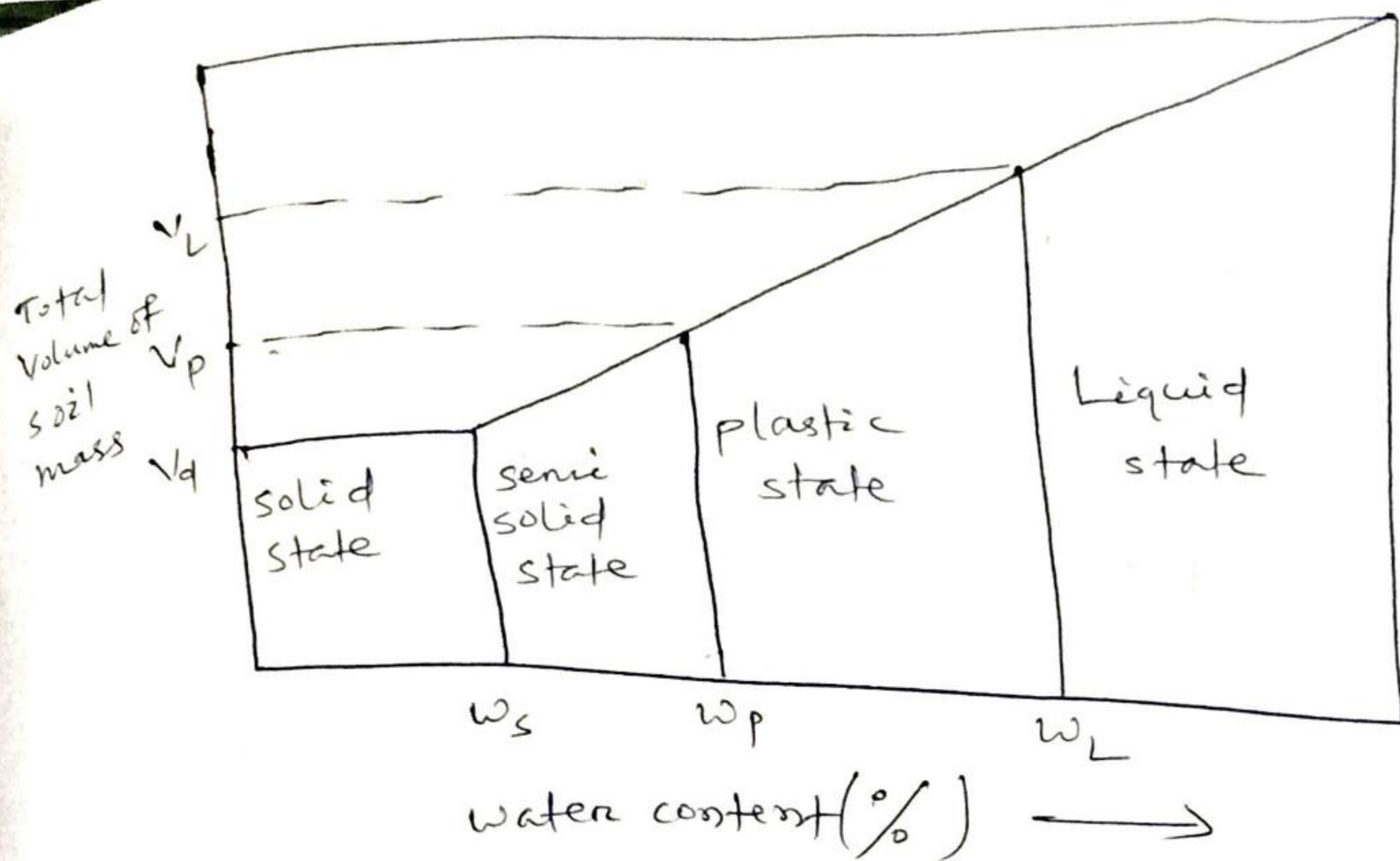
It is meant the relative ease with which the soil can be deformed. Consistency denotes the degree of firmness of soil which may be termed as soft, firm, stiff or hard.

Fine grained soil may be mixed with water to form a plastic paste which can be moulded into any form by pressure.

A scientist Atterberg divided the entire range from liquid to solid state into four stages.

- i) Liquid state
- ii) plastic state
- iii) semi-solid state
- iv) solid state

Hence the consistency limits are the water contents at which the soil mass passes from one state to the next.



V_L = volume of liquid
 V_p = volume of plastic
 V_d = Dry volume

Liquid limit (w_L)

Liquid limit is the water content corresponding to the arbitrary limit between liquid and plastic state.

plastic limit (w_p)

It is the water content corresponding to the arbitrary limit between liquid and plastic state.

Shrinkage limit (w_s)

It is the water content corresponding to the arbitrary limit between semisolid and solid state.

Plasticity Index (I_p)

The range of consistency within which a soil exhibits plastic properties is called plastic range and is indicated by plasticity index.

Plasticity index is defined as the numerical difference between liquid limit and plastic limit.

$$\text{Mathematically } I_p = W_L - W_p$$

Plasticity It is defined as that property of a soil which allows it to be deformed rapidly without rupture, without elastic rebound and without volume change.

Consistency index (I_c)

It is defined as the ratio of liquid limit minus natural water content to the plasticity index of soil -

$$\text{Mathematically } I_c = \frac{W_L - W}{I_p}$$

$$\Rightarrow I_c = \frac{W_L - W}{W_L - W_p}$$

Liquidity index (I_L)

It is defined as the ratio of natural water content of a soil minus its plastic limit to its plasticity index.

$$\text{Mathematically } I_L = \frac{W - W_p}{I_p}$$

$$\Rightarrow I_L = \frac{W - W_p}{W_L - W_p}$$

1) If liquidity index is 1, then

$$1 = \frac{W - W_p}{W_L - W_p}$$

$$\Rightarrow W - W_p = W_L - W_p$$

$$\Rightarrow W = W_L$$

Soil is at liquid limit

$$2) \text{ If } I_L = 0, \quad 0 = \frac{W - W_p}{W_L - W_p}$$

$$\Rightarrow W - W_p = 0 \Rightarrow W = W_p$$

Soil is at plastic limit

$$3) \text{ If } 0 < I_L < 1$$

Soil is in plastic state

$$4) \text{ If } I_L > 1$$

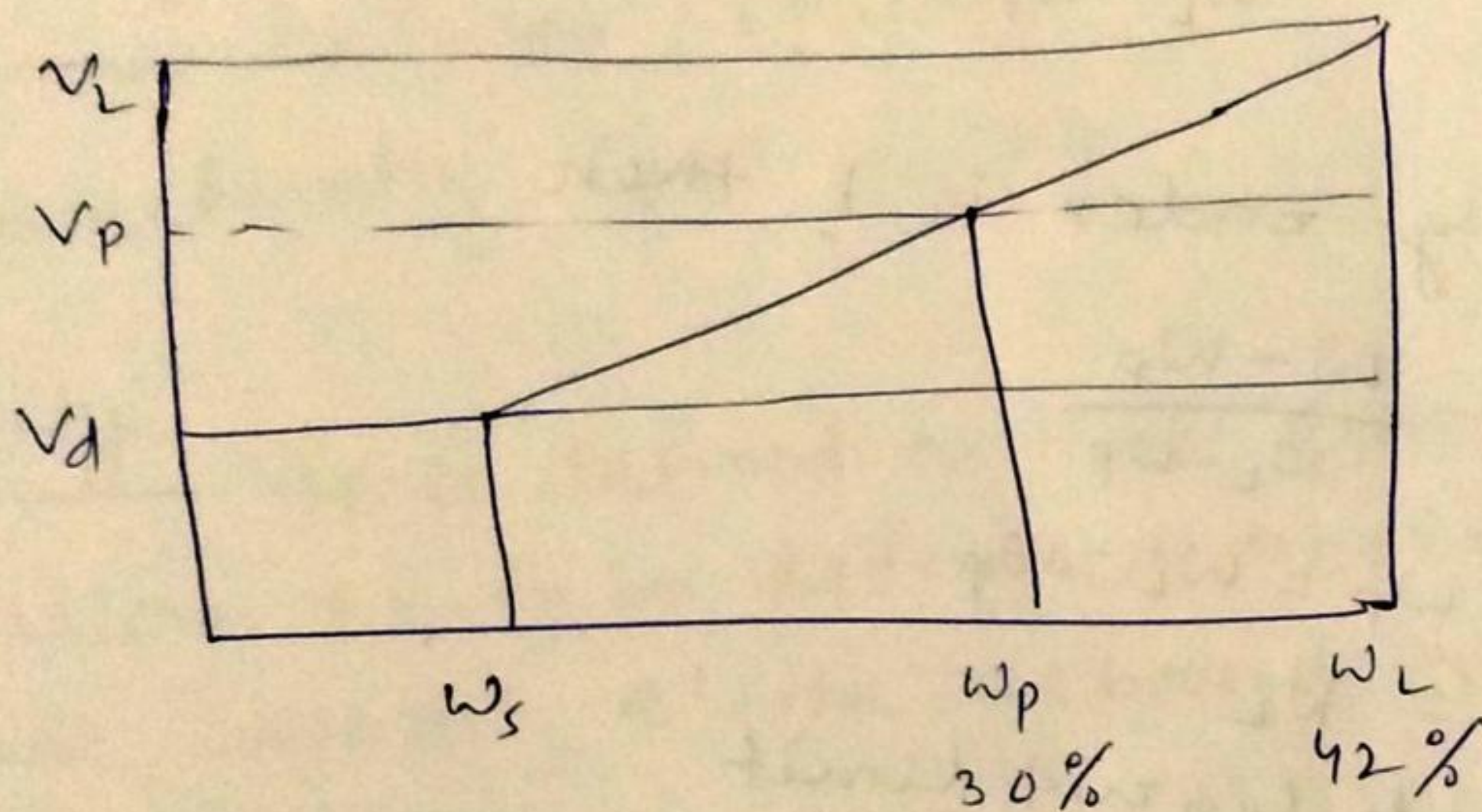
Soil is in liquid state

$$5) \text{ If } I_L < 0$$

Soil is in solid or semisolid state

Q.1 The plastic limit and liquid limit of a soil are 30% and 42% respectively. The percentage volume change from liquid limit to dry state is 35% of the dry volume. Similarly the percentage change in volume from plastic limit to dry state is 22% of the dry volume. Determine shrinkage limit?

Ans



Given data $w_L = 42\%$, $w_p = 30\%$

$$V_L - V_d = 0.35 V_d$$

$$\Rightarrow V_L = V_d + 0.35 V_d = 1.35 V_d \quad \text{--- (1)}$$

$$V_p - V_d = 0.22 V_d$$

$$\Rightarrow V_p = V_d + 0.22 V_d = 1.22 V_d \quad \text{--- (2)}$$

From figure $\frac{V_L - V_p}{w_L - w_p} = \frac{V_L - V_d}{w_L - w_s}$

$$\Rightarrow \frac{1.35 V_d - 1.22 V_d}{0.42 - 0.30} = \frac{1.35 V_d - V_d}{0.42 - w_s}$$

$$\Rightarrow \frac{0.13 V_d}{0.12} = \frac{0.35 V_d}{0.42 - w_s}$$

$$\Rightarrow \frac{0.13}{0.12} = \frac{0.35}{0.42 - w_s} \quad \Rightarrow w_s = 0.0969$$

$$\Rightarrow w_s = 9.69\%$$

Flow index

The slope of flow curve is termed as flow index which represents the rate of loss of the shear strength of soil with increase in water content.

$$\text{slope of flow curve} = \frac{w_1 - w_2}{\log N_1 - \log N_2}$$

$$\text{flow index} = \left| \frac{w_1 - w_2}{\log N_2 - \log N_1} \right|$$

$$\Rightarrow I_f = \frac{w_1 - w_2}{\log \left(\frac{N_2}{N_1} \right)}$$

Higher will be the value of flow index for a particular soil lower will be its shear strength corresponding to its water content.

Toughness index (I_T)

It is defined as the ratio of (I_p) plasticity index of soil to the flow index (I_f).

$$\text{Mathematically } I_T = \frac{I_p}{I_f}$$

Q.1 The following data on consistency limits are available for two soils A and B.

	<u>soil (A)</u>	<u>soil (B)</u>
1) plastic limit	16%	19%
2) Liquid limit	30%	52%
3) flow index	11	6
4) Natural water content	32%	40%

Find which soil is (a) more plastic

(b) better foundation material

(c) better shear strength as a function of water content

(d) better shear strength at plastic limit

Ans (a) Plasticity Index I_p for soil A

$$= 30 - 16 = 14$$

plasticity Index I_p for soil B

$$= 52 - 19 = 33$$

Since plasticity Index of soil 'B' is greater than A, hence soil B is more plastic

(b) Consistency index I_c for soil A

$$= \frac{w_L - w}{I_p} = \frac{30 - 32}{24} = -0.083$$

Consistency index I_c for soil B

$$= \frac{w_L - w}{I_p} = \frac{52 - 40}{33}$$

$$= 0.363$$

Since consistency index for soil A is negative and soil B is positive. Hence soil B is suitable for foundation.

(c) Flow index I_f for soil A is 11 and flow index I_f for soil B is 6.

Since flow index of soil B is lesser than of soil A. Hence soil B has better shear strength as a function of water content.

(d) Toughness index I_T for soil A

$$= \frac{I_p}{I_f} = \frac{24}{11} = 2.18$$

Toughness index I_T for soil B

$$= \frac{I_p}{I_f} = \frac{33}{6} = 5.5$$

Since toughness index of soil B is greater than that of A.

Hence soil B has a better shear strength at plastic limit.

Textural classification of soils

Soils occurring in nature are composed of different percentage of sand, silt and clay size particles. Soil classification of composite soils exclusively based on the particle size distribution \rightarrow known as ~~texture~~ textural classification.

Group index of soil

The group index of soil depends upon

i) The amount of material passing the 75 micron '15' sieve.

ii) The liquid limit

iii) The plastic limit

$$\text{Group index} = 0.2a + 0.005ac + 0.01bd$$

where a = that portion of percentage passing 75 micron sieve greater than 35 and not exceeding 75 expressed as a whole number (0 to 40)

b = that portion of percentage passing 75 micron sieve greater than 15 and not exceeding 55 expressed as a whole number (0 to 40)

c = that portion of numerical liquid limit greater than 40 and not exceeding 60 expressed as positive whole number (0 to 20).

d = that portion of numerical plasticity index greater than 10 and not exceeding 30 expressed as positive whole number (0 to 20).

if the maximum values of a, b, c and d are taken

i.e.

$$\begin{aligned} a &= 40 \\ b &= 40 \\ c &= 20 \\ d &= 20 \end{aligned}$$

$$\begin{aligned} \text{Then } G.I. &= 0.2a + 0.005ac + 0.01bd \\ &= 0.2 \times 40 + 0.005 \times 40 \times 20 + 0.01 \times 40 \times 20 \\ &= 8 + 4 + 8 = 20 \end{aligned}$$

<u>NO</u>	<u>Value of G.I.</u>	<u>Soil condition</u>
1	0	Excellent
2	1	Good
3	2 to 4	Fair
4	5 to 9	poor
5	10 to 20	very poor

Q.1 Calculate the group index of a soil with following particulars,

Percentage passing 75 micron sieve = 60

Liquid limit = 30

Plasticity Index = 12

Ans

$$G.I. = 0.2a + 0.005ac + 0.01bd$$

$$a = 60 - 35 = 25$$

$$b = 55 - 15 = 40$$

$$c = 0$$

$$d = 12 - 10 = 2$$

$$G.I. = 0.2 \times 25 + 0.005 \times 25 \times 0 + 0.01 \times 40 \times 2$$

$$= 5.80 \approx 6$$

Hence soil is poor quality.

Q.2 On sieve analysis it has been found that the soil contains 32 percent material passing 75 micron sieve. If the liquid limit and plastic limit of soil are respectively 42.5 percent and 26.7 percent, determine the group index of soil?

Ans $G.I. = 0.2a + 0.005ac + 0.01bd$
 $a = 0, b = 32 - 15 = 17$
 $c = 42.5 - 40 = 2.5$
 $P.I. = 42.5 - 26.7 = 15.8$
 $d = 15.8 - 10 = 5.8$
 $G.I. = 0.2 \times 0 + 0.005 \times 0 \times 2.5 + 0.01 \times 17 \times 5.8$
 $= 0.986 \approx 1$ (soil is good quality)

Q.3 On a laboratory test of soil sample obtained that 56 percent of material passing through 75 micron sieve. If the liquid limit and plastic limit are 36 percent and 23 percent respectively. Determine the group index of soil?

Ans $a = 56 - 35 = 21$
 $b = 56 - 15 = 41$
 $c = 0$
 $P.I. = 36 - 23 = 13$
 $d = 13 - 10 = 3$
Hence $G.I. = 0.2 \times 21 + 0.005 \times 21 \times 0 + 0.01 \times 40 \times 3$
 $= 5.4$
Hence soil is poor quality.

Permeability of soil

A soil mass is composed of small solid particles which is called soil grains. The soil masses arrange themselves in such a way that an empty space exists which is called voids. These voids are interconnected tube like structure. Water flows from voids of higher potential to lower potential.

The more narrow or irregular void than water cannot flow easily.

The regular and open voids are present then water flows easily.

Permeability

It is the ease with which water can flow through soils. The property of soil which permits a liquid to flow through its voids is called permeability.

Large soil particles have large volume of voids. Hence better connectivity of voids.

Gravels have large volume of voids. Hence higher water will flow. Therefore it has high permeability.

Soils

Gravels _____

Sand _____

Silt _____

Clay _____

Permeability of soils (cm/sec)
 > 1

$1 - 10^{-3}$

$10^{-3} - 10^{-7}$

$< 10^{-7}$

Gravels have more permeable where as clay have least.

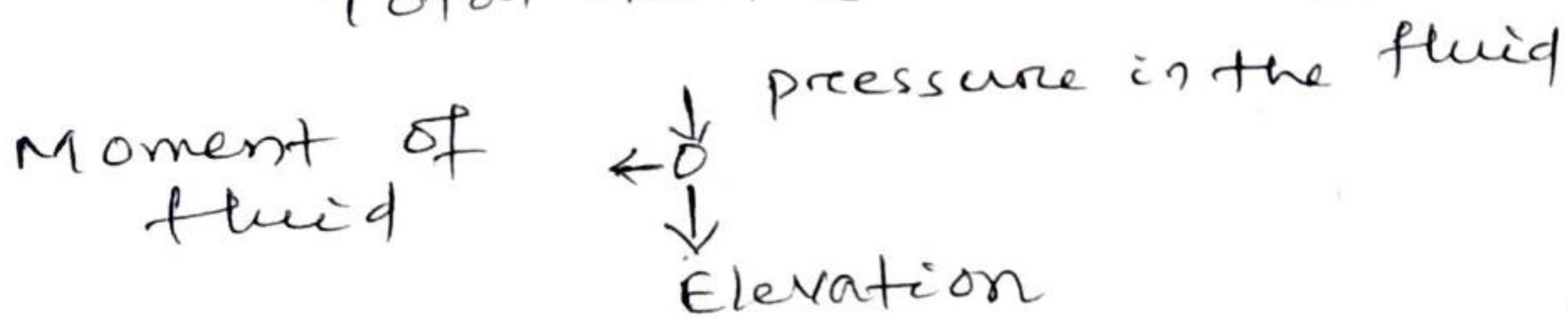
Clay soils have high void ratio. It has high volume of voids because of flocculated structure. These clay particles are very small soil particles, these are poorly connected to pores. Hence clay form irregular tube. Hence even after having large amount of voids clays are less permeable.

→ When soil has low permeability of soil it is called impervious.

Water can flow from one point to another when there is a difference of hydraulic head. Hydraulic head is mechanical energy available at any point in water.

From Bernoulli principle,

Total head (Total energy) =



Moment of fluid = velocity head

pressure in the fluid = pressure head

Elevation of fluid = Elevation head

Hence total head = velocity head + pressure head + Elevation head

Datum Any reference line or plane. Total head is zero at height (0m). When water flows through the soil then its velocity is small. Therefore velocity head is zero. Water flows from high energy region to low energy region.

Darcy's Law

The velocity of flow of liquid between two points in the soil is directly proportional to the hydraulic gradient applied to it.

Velocity of flow \propto Hydraulic gradient

$$v \propto i$$
$$\Rightarrow v = k i \quad (k = \text{a permeability constant})$$

v = velocity of flow (discharge velocity)

i = hydraulic gradient

k = coefficient of permeability

$$k = \frac{v}{i}$$

When $i = 1$, $k = v$

Hence the coefficient of permeability is the velocity of flow of liquid inside the soil if the hydraulic gradient is unity.

Unit of k is m/sec.

$$i = \text{Dimensionless} = \frac{\Delta h}{L}$$

$$Q = \text{Discharge} = AV$$

where $A =$ Area of cross-section

$v =$ velocity of flow

$$Q = A \times k \dot{i}$$

$$\Rightarrow \boxed{Q = k \dot{i} A}$$

$$\boxed{Q = KA \left(\frac{\Delta h}{L} \right)}$$

Constant head permeability test

Water flows from the overhead tank consisting of three tubes,

- i) the inlet tube
- ii) the overflow tube
- iii) the outlet tube

The constant hydraulic gradient ' i ' causing the flow is the head h divided by length ' L ' of the sample.

If ' Q ' is the total quantity of flow in a time interval ' t ' we have from Darcy's law,

$$Q = \frac{Q}{t} = k \dot{i} A$$

$$\Rightarrow k = \frac{Q}{t} \times \left(\frac{1}{i A} \right)$$

$$\text{But } i = \frac{h}{L}$$

$$\text{Hence } K = \left(\frac{Q}{t}\right) \left(\frac{L}{h}\right) \left(\frac{1}{A}\right)$$

where A = cross-sectional area of the specimen sample

Q.1 A constant head permeability test was run on a sand sample 16 cm in length and 60 cm^2 in cross-sectional area. Under a constant head of 30 cm the discharge was found to be 45 cm^3 in 18 seconds. Calculate coefficient of permeability and discharge velocity.

Ans

$$L = 16 \text{ cm}$$
$$A = 60 \text{ cm}^2$$
$$h = 30 \text{ cm}$$

$$\frac{Q}{t} = \frac{45}{18} \frac{\text{cm}^3}{\text{sec}}$$

$$K = \left(\frac{Q}{t}\right) \frac{L}{h} \left(\frac{1}{A}\right)$$
$$= \frac{45}{18} \times \frac{16}{30} \times \frac{1}{60} = 2.22 \times 10^{-2} \text{ cm/sec}$$

Discharge velocity $V = K \cdot i = K \left(\frac{h}{L}\right)$

$$= 2.22 \times 10^{-2} \times \frac{30}{16}$$
$$= 4.17 \times 10^{-2} \text{ cm/sec}$$

Q-2 Calculate the coefficient of permeability of a soil sample 6cm in height and 50cm^2 in cross-sectional area. If a quantity of water equal to 430ml passed down in 10 minutes under an effective constant head of 40cm. Also calculate discharge velocity?

Ans

$$Q = 430\text{ml}$$
$$t = 10\text{min} = 10 \times 60 = 600\text{sec}$$
$$A = 50\text{cm}^2$$
$$L = 6\text{cm}$$
$$h = 40\text{cm}$$

$$k = \left(\frac{Q}{t}\right) \left(\frac{L}{h}\right) \left(\frac{1}{A}\right)$$
$$= \frac{430}{600} \times \frac{6}{40} \times \frac{1}{50} = 2.15 \times 10^{-3} \text{cm/sec}$$

Then discharge velocity

$$v = kv = k \times \frac{h}{L}$$
$$= 2.15 \times 10^{-3} \times \frac{40}{6}$$
$$= 1.435 \times 10^{-2} \text{cm/sec}$$

Falling head permeability test

The constant head permeability test is used for coarse grained soil only where a reasonable discharge can be collected in a given time. However the falling head test is used for relatively less permeable soils which is given by discharge is small.

A stand pipe of known cross-sectional area 'a' is fitted over the permeameter and water is allowed to run down. The water level in the stand pipe constantly falls as water flows. The head at any time instant 't' is equal to the difference in the water level in the stand pipe and bottom tank.

Let h_1 and h_2 be the heads at time interval t_1 and t_2 ($t_2 > t_1$).

Let 'h' be the head at any intermediate time interval 't' and $(-dh)$ be the change in the smaller time interval 'dt'. Minus sign has been used since 'h' decreases as 't' increases.

Hence from Darcy's Law the rate of flow 'q' is given by

$$q = \frac{(-dh \cdot a)}{dt} = K \cdot i \cdot A$$

where $i =$ hydraulic gradient at time
 $i = h/L$

$$\text{Hence } K \frac{h}{L} A = \left(\frac{-dh}{dt} \right) a$$

$$\Rightarrow \frac{AKh}{La} = \left(\frac{-dh}{dt} \right)$$

$$\Rightarrow \frac{AK}{aL} dt = - \left(\frac{dh}{h} \right) \quad \text{--- (1)}$$

Integrating equation (1) we get

$$\frac{AK}{aL} \int_{t_1}^{t_2} dt = \int_{h_1}^{h_2} - \left(\frac{dh}{h} \right)$$

$$\Rightarrow \frac{AK}{aL} [t]_{t_1}^{t_2} = - \left[\log_e h \right]_{h_1}^{h_2}$$

$$\Rightarrow \frac{AK}{aL} (t_2 - t_1) = \log_e h_1 - \log_e h_2$$

Denoting $(t_2 - t_1) = t$ we get

$$\frac{AK}{aL} (t) = \log_e \left(\frac{h_1}{h_2} \right)$$

$$\Rightarrow \boxed{K = \frac{aL}{At} \cdot 2.3 \log_{10} \left(\frac{h_1}{h_2} \right)}$$

Q.1 In a falling head permeameter test the initial head ($t=0$) is 40cm. The head drops by 5cm in 10 minutes. Calculate the time required to run the test for the final head to be at 20cm. If the sample is 6cm height and 50cm^2 in cross-sectional area; Calculate coefficient of permeability taking area of stand pipe = 0.5cm^2

Ans

$$h_1 = 40\text{cm}$$

$$h_2 = 40 - 5 = 35\text{cm}$$

$$t = 10\text{min} = 10 \times 60 = 600\text{sec}$$

$$a = 0.5\text{cm}^2$$

$$A = 50\text{cm}^2$$

$$L = 6\text{cm}$$

$$\begin{aligned} \text{permeability } K &= 2.3 \frac{aL}{At} \log_{10} \left(\frac{h_1}{h_2} \right) \\ &= 2.3 \times \frac{0.5 \times 6}{50 \times 600} \log_{10} \left(\frac{40}{35} \right) \\ &= 1.335 \times 10^{-5} \text{ cm/sec} \end{aligned}$$

Q.2 In a falling head permeameter test on a silty clay sample, the following results were obtained.
 sample length = 12mm, sample diameter = 80mm
 initial head = 1200mm, final head = 400mm.
 time for fall in head 6 minutes, stand pipe diameter = 4mm. Find the coefficient of permeability of the soil in cm/sec.

Ans

$$\begin{aligned} L &= 12 \text{ cm} \\ D &= 80 \text{ mm} = 8 \text{ cm} \\ A &= \frac{\pi}{4} (D)^2 = \frac{\pi}{4} \times (8)^2 = 50.265 \text{ cm}^2 \\ d &= 4 \text{ mm} = 0.4 \text{ cm} \\ a &= \frac{\pi}{4} (d)^2 = \frac{\pi}{4} (0.4)^2 = 0.1257 \text{ cm}^2 \\ t &= 6 \text{ min} = 6 \times 60 = 360 \text{ sec} \\ h_1 &= 1200 \text{ mm} = 120 \text{ cm} \\ h_2 &= 400 \text{ mm} = 40 \text{ cm} \\ K &= 2.303 \frac{aL}{At} \log_{10} \left(\frac{h_1}{h_2} \right) \\ &= 2.303 \times \frac{0.1257 \times 12}{50.265 \times 360} \log_{10} \left(\frac{120}{40} \right) \\ &= 9.159 \times 10^{-5} \text{ cm/sec} \end{aligned}$$

Shear strength

The shear strength of a material is the greatest stress it can sustain.

The safety of any geotechnical structure is dependent on the strength of the soil. If the soil fails the structure founded on it can collapse.

The shear strength is the capacity of a material to resist the internal and external forces which slide past each other.

Significance of shear strength

Engineers must understand the nature of shearing resistance in order to analyze soil stability problems such as bearing capacity, slope stability,

Shear strength in soils

The shear strength of a soil is its resistance to shearing stresses. It is a measure of the soil resistance to deformation by continuous displacement of its individual soil particles. Shear strength in soil depends primarily on interactions between particles.

→ Shear failure occurs when the stresses between the particles are such that they slide or roll past each other.

Components of shear strength of soils

Soils derives its shear strength from two sources.

- cohesion between particles.
- frictional resistance and interlocking between particles.

Cohesion

It is a force of attraction between the particles binding them together. Cohesion is present in clays and silts but is normally absent in sands and gravels. It is represented as 'c'.

Angle of repose is determined by

- particle size
- particle shape
- shear strength

Stresses

Gravity generates stresses in the ground at different points. Stresses on a plane at a given point is viewed in terms of two components.

- Normal stress - It acts normally to the plane and tends to compress soil grains towards each other.
- Shear stress - It acts tangential to the plane and tends to slide grains relative to each other.

Factors influencing shear strength

The shear strength is affected by.

- soil compaction - It includes mineralogy, grain size and grain size distribution, shape of particles, pore fluid type.
- Initial state - It can be describe by terms such as loose, dense, over consolidated, normally consolidated, stiff, soft etc.
- structure - It refers to the arrangement of particles within the soil mass, the manner in which the particles are packed or distributed.

Mohr - Coulomb failure theory

$$\tau_f = c + G \tan \phi$$

where c = cohesion
 ϕ = angle of internal friction

This theory states that a material fails because of critical combination of normal stress and shear stress.

where τ_f = shear stress

G = normal stress

Following points are essential in Mohr's column theory.

- 1) Materials fail essentially by shear. The critical shear causing failure depends upon the properties of the material as well as on normal stress on the failure plane.
- 2) The ultimate strength of the material is determined by the stresses on the plane of shear.
- 3) When the material is subjected to three dimensional principal stresses ($\sigma_1, \sigma_2, \sigma_3$) the intermediate principal stress does not have any influence on the strength of material.

Mohr's stress circle

Through a point in a loaded soil mass the stress component on each plane depends upon the direction of plane. There exists three typical planes mutually orthogonal to each other. Here the stress is wholly normal and no shear stress acts. These planes are called principal planes and the normal stresses acting on these planes are called principal stresses.

In order of decreasing magnitude of normal stress these planes are called major, intermediate and minor planes.

The corresponding normal stresses on them are called major principal stress σ_1 , intermediate principal stress σ_2 and minor principal stress σ_3 .

The normal stress 'G' and shearing stress 'τ' on any plane MN inclined at an angle α with x-direction.

$$G = \frac{\sigma_y + \sigma_x}{2} + \left(\frac{\sigma_y - \sigma_x}{2} \right) \cos 2\alpha + \tau_{xy} \sin 2\alpha \quad \text{--- (1)}$$

and

$$\tau = \frac{\sigma_y - \sigma_x}{2} \sin 2\alpha - \tau_{xy} \cos 2\alpha \quad \text{--- (2)}$$

Where σ_y and σ_x = Normal stresses on planes perpendicular to y and x axes and $(\sigma_y \neq \sigma_x)$,

$\tau_{xy} = \tau_{yx}$ = Shear stress on these two planes.

Squaring equⁿ (1) and equⁿ (2) and adding we get,

$$\left(G - \frac{\sigma_y + \sigma_x}{2} \right)^2 + \tau^2 = \left(\frac{\sigma_y - \sigma_x}{2} \right)^2 + \tau_{xy}^2$$

Q.1 A point in a strained material is subjected to two mutually perpendicular tensile stresses of 200 N/mm^2 and 100 N/mm^2 . Determine the intensities of normal, shear, and resultant stresses on a plane inclined at 30° with the axis of minor tensile stresses.

Ans $\sigma_y = 200 \text{ N/mm}^2$

$\sigma_x = 100 \text{ N/mm}^2$

$\alpha = 30^\circ$

$$\sigma_n = \left(\frac{\sigma_y + \sigma_x}{2} \right) + \left(\frac{\sigma_y - \sigma_x}{2} \right) \cos 2\alpha$$

$$= \left(\frac{200 + 100}{2} \right) + \left(\frac{200 - 100}{2} \right) \cos(2 \times 30^\circ)$$

$$= 125 \text{ N/mm}^2$$

$$\text{Shear stress } \tau = \frac{\sigma_y - \sigma_x}{2} \sin 2\alpha$$

$$= \frac{(200 - 100)}{2} \sin(2 \times 30^\circ)$$

$$= 50 \sin 60^\circ = 43.3 \text{ N/mm}^2$$

$$\text{Resultant stress on inclined plane } \sigma_R = \sqrt{\sigma_n^2 + \tau^2}$$

$$= \sqrt{(125)^2 + (43.3)^2} = 132.3 \text{ N/mm}^2$$

Q.2 The stresses on a point are 150 N/mm^2 and 50 N/mm^2 both tensile. Find the intensities of normal, shear and resultant stresses on a plane inclined at an angle of 55° with the axis of major tensile stress.

Ans

$$\sigma_y = 150 \text{ N/mm}^2$$

$$\sigma_x = 50 \text{ N/mm}^2$$

$$\alpha = 55^\circ$$

$$\text{Normal stress } \sigma_n = \frac{\sigma_y + \sigma_x}{2} - \frac{\sigma_y - \sigma_x}{2} \cos 2\alpha$$

$$= \frac{150 + 50}{2} - \frac{150 - 50}{2} \cos(2 \times 55^\circ)$$

$$= 100 - 50 \cos(110^\circ) = 117.1 \text{ N/mm}^2$$

$$\text{Shear stress } \tau = \frac{\sigma_y - \sigma_x}{2} \sin 2\alpha$$

$$= \frac{150 - 50}{2} \sin(2 \times 55^\circ)$$

$$= 47 \text{ N/mm}^2$$

Resultant stress on the inclined plane

$$\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$$

$$= \sqrt{(117.1)^2 + (47)^2}$$

$$= 126.2 \text{ N/mm}^2$$

Earth pressure

Lateral earth pressure is the pressure that soil exerts in the horizontal direction. The lateral earth pressure is important because it affects the consolidation behaviour and strength of soil.

The coefficient of lateral earth pressure 'K' is defined as the ratio of horizontal effective stress σ'_h to the vertical effective stress σ'_v .

$$K = \frac{\sigma'_h}{\sigma'_v}$$

K for a particular soil deposit is a function of soil properties and stress history.

- The minimum stable value of K is called the active earth pressure coefficient ' k_a '. For example the retaining wall is moving away from soil.
- The maximum stable value of K is called the passive earth pressure coefficient ' k_p '. For example the vertical flow that is pushing the soil horizontally.
- For a level ground deposit with zero lateral strain in the soil at rest, coefficient of lateral earth pressure ' k_0 ' is obtained.

Lateral earth pressure at rest

$$k_0 = 1 - \sin \phi$$

Where k_0 = coefficient of lateral earth pressure at rest.

ϕ = angle of cohesion or internal friction
 It is noted that the earth pressure at rest (P_0) is always greater than active earth pressure (P_a) but lesser than passive earth pressure (P_p).

$$P = \frac{1}{2} K \gamma H^2$$

Where K = coefficient of earth pressure

γ = unit weight of soil.

H = Height

$$P_0 = \frac{1}{2} K_0 \gamma H^2 \rightarrow \text{at rest}$$

$$P_a = \frac{1}{2} K_a \gamma H^2 \rightarrow \text{Active}$$

$$P_p = \frac{1}{2} K_p \gamma H^2 \rightarrow \text{Passive}$$

Q.1 A rigid retaining wall 6m high is restrained from yielding. The backfill consists of cohesionless soil having $\phi = 26^\circ$ and $\gamma = 19 \text{ kN/m}^3$. Compute the total earth pressure per mt. length of the wall.

Ans

$$\phi = 26^\circ, \gamma = 19 \text{ kN/m}^3$$

$$H = 6 \text{ mt}$$

$$K_0 = 1 - \sin \phi = 1 - \sin 26^\circ = 0.5616$$

$$P_0 = \frac{1}{2} K_0 \gamma H^2$$

$$= \frac{1}{2} \times 0.5616 \times 19 \times (6)^2$$

$$= 192.1 \text{ kN/m length of wall}$$

Types of earth pressure

- Active earth pressure
- Passive earth pressure
- Earth pressure at rest

Active earth pressure

The minimum value of lateral earth pressure exerted by a soil on a structure occurring when the soil is allowed to yield sufficient to cause internal shearing resistance along a potential failure surface.

Passive earth pressure

When the wall moves towards the backfill there is an increase in the pressure on the wall and this increase continues until a maximum value has reached.

After which there is no increase in the pressure and the value will become constant. This kind of pressure is known as passive earth pressure.

Earth pressure at rest

Under conditions where there is no lateral strain within the ground mass the value of lateral soil pressure is called lateral earth pressure at rest (K_0).

It is also defined as the neutral lateral earth pressure or the lateral earth pressure at consolidated equilibrium.

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi}$$

Q.1 Compute the intensities of active and passive earth pressure at a depth of 8m in dry cohesionless sand with an angle of internal friction of 30° and unit weight of 18 kN/m^3 . Calculate P_a and P_p if the water level rises to ground level.

Ans $\gamma = 18 \text{ kN/m}^3$

$H = 8 \text{ m}$

$\phi = 30^\circ$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = \frac{1 - 1/2}{1 + 1/2}$$
$$= \frac{1}{3}$$

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi}$$
$$= \frac{1 + \sin 30^\circ}{1 - \sin 30^\circ}$$
$$= \frac{1 + 1/2}{1 - 1/2} = \frac{3/2}{1/2} = 3$$

$$P_a = K_a \gamma H = \frac{1}{3} \times 18 \times 8 = 48 \text{ kN/m}^2$$

$$P_p = K_p \gamma H$$
$$= 3 \times 18 \times 8$$
$$= 432 \text{ kN/m}^2$$

Active earth pressure Rankine's theory

Rankine's theory of lateral earth pressure is applied to uniform cohesionless soils only.

Following are the assumptions,

- i) The soil mass is semi-infinite, homogeneous, dry and cohesionless.
- ii) The ground surface is a plane which may be horizontal or inclined.
- iii) The back of the wall is vertical or smooth.
- iv) The wall yields about the base.

The ratio of the horizontal stress G_h to the vertical stress G_v is called coefficient of earth pressure 'k'.

When the soil is in the active state of plastic equilibrium,

$$G_h = G_3$$

and $G_v = G_1$

$$\text{Then } k_a = \frac{G_h}{G_v} = \frac{1}{\tan^2\left(45^\circ + \frac{\phi}{2}\right)}$$
$$= \cot^2\left(45^\circ + \frac{\phi}{2}\right) = \frac{1 - \sin\phi}{1 + \sin\phi}$$

Similarly in passive state

$$G_h = G_1$$

and $G_v = G_3$

$$\text{Then } k_p = \frac{G_h}{G_v} = \tan^2\left(45^\circ + \frac{\phi}{2}\right) = \frac{1 + \sin\phi}{1 - \sin\phi}$$

When the soil is at elastic equilibrium i.e. at rest the ratio of horizontal to vertical stress is called coefficient of earth pressure at rest.

$$\text{i.e. } k_0 = \frac{G_h}{G_v}$$

Passive earth pressure for cohesionless backfill

In case of passive state of plastic equilibrium the lateral pressure is the major principal stress while the vertical pressure is the minor principal stress.

$$\text{Thus } \sigma_h = P_p = \sigma_1 \\ \sigma_v = \sigma_3 = \gamma Z$$

Substituting these in the principal stress relationship $\sigma_1 = \sigma_3 \tan^2 \alpha$

$$\Rightarrow P_p = \gamma Z \tan^2 \alpha$$

$$\Rightarrow \boxed{P_p = K_p \gamma Z}$$

Where P_p = passive earth pressure intensity
 K_p = Rankine's ~~ear~~ coefficient of passive earth pressure

$$K_p = \tan^2 \alpha = N_\phi = \frac{1 + \sin \phi}{1 - \sin \phi} = \frac{1}{K_a}$$

$$\text{Also ratio } \frac{K_p}{K_a} = \frac{\tan^2(45^\circ + \phi/2)}{\cot^2(45^\circ + \phi/2)}$$

$$\begin{aligned} \Rightarrow \frac{K_p}{K_a} &= \tan^2(45^\circ + \frac{\phi}{2}) \times \frac{1}{\cot^2(45^\circ + \frac{\phi}{2})} \\ &= \tan^2(45^\circ + \frac{\phi}{2}) \times \tan^2(45^\circ + \frac{\phi}{2}) \\ &= \tan^4(45^\circ + \frac{\phi}{2}) \end{aligned}$$

$$\text{If } \phi = 30^\circ$$

$$\frac{K_p}{K_a} = \tan^2\left(45^\circ + \frac{30^\circ}{2}\right) = \tan^2(60^\circ)$$
$$= (\tan 60^\circ)^2 = (\sqrt{3})^2 = 9$$

$$\text{Hence } \boxed{K_p = 9K_a}$$

Similarly If $\phi = 30^\circ$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ}$$

$$= \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$$

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi} = \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}} = 3$$

$$\frac{K_p}{K_a} = \frac{3}{\frac{1}{3}} = 3 \times 3 = 9$$

$$\Rightarrow \boxed{K_p = 9K_a}$$

The distribution of passive earth pressure is triangular with maximum value of $K_p \gamma H$ at the base of retaining wall of height H .

The total pressure ' P_p ' for a depth H is

$$\text{given by } P_p = \int_0^H K_p \gamma z \, dz$$

$$P_p = K_p \gamma \left(\frac{z^2}{2}\right)_0^H$$

$$P_p = \frac{1}{2} K_p \gamma H^2$$

Q.2 A retaining wall 4m high has a smooth vertical back. The backfill has a horizontal surface in level with the top of the wall. There is uniformly distributed surcharge load of 36 kN/m^2 intensity over the backfill. The unit weight of backfill is 18 kN/m^3 . Its angle of shearing resistance is 30° and cohesion is zero. Determine the magnitude and point of application of active pressure per metre length of the wall.

Ans

$$k_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ}$$

$$= \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = \frac{1}{3}$$

The lateral pressure due to surcharge is given by $P_1 = k_a q = \frac{1}{3} \times 36 = 12 \text{ kN/m}^2$

The pressure intensity due to the backfill at a depth $H = 4 \text{ m}$ is given by

$$P_2 = k_a \gamma H = \frac{1}{3} \times 18 \times 4 = 24 \text{ kN/m}^2$$

$$q = \text{Surcharge load} = 36 \text{ kN/m}^2$$

$$\gamma = 18 \text{ kN/m}^3$$

$$\phi = 30^\circ$$

$$H = 4 \text{ m}$$

The total pressure intensity at the base of the wall is given by

$$P_a = P_1 + P_2 = 12 + 24 = 36 \text{ kN/m}^2$$

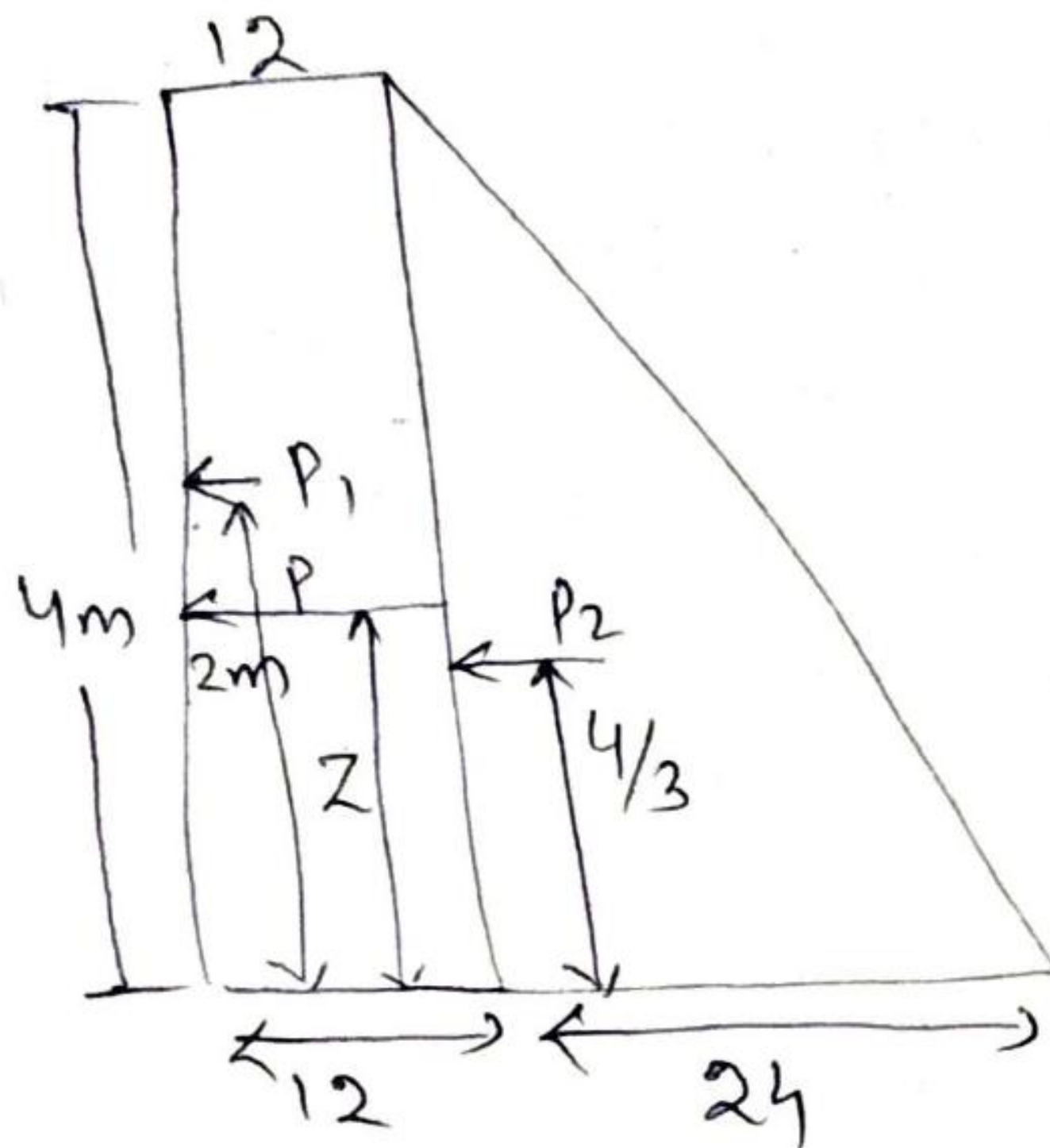


Figure shows the pressure distribution for P_1 and P_2 .

The resultant total pressure P_1 due to intensity P_1 is given by

$$P_1 = P_1 \times H = 12 \times 4 = 48 \text{ kN/m}$$

acting at $\frac{4}{2} = 2 \text{ m}$ from the base.

The resultant total pressure P_2 due to intensity P_2 is given by

$$P_2 = \frac{1}{2} P_2 \times H = \frac{1}{2} \times 24 \times 4 = 48 \text{ kN/m}$$

acting at $\frac{1}{3} \times 4 = 1.33 \text{ m}$ from the base.

Hence the resultant P acts at a distance \bar{z} above the base given by taking the moments about the base,

$$\bar{z} = \frac{48 \times 2 + 48 \times \frac{4}{3}}{48 + 48} = 1.67 \text{ m}$$

Surcharge loads acting on retaining walls are additional vertical loads that used to the backfill soil above the top of the wall.

The load supporting above the level of the top of a retaining wall.

Earth pressure at rest

The earth pressure at rest, exerted on the back of a rigid, unyielding retaining structure can be calculated using theory of elasticity, assuming soil to be semi-infinite, homogeneous, elastic and isotropic.

Consider an element of soil at a depth 'z' being acted upon by vertical stress G_v and horizontal stress G_h . There will be no shear stress. The lateral strain ϵ_h in the horizontal direction is given by.

$$\epsilon_h = \frac{1}{E} [G_h - \mu (G_h + G_v)]$$

The earth pressure at rest corresponding to the condition of zero lateral strain ($\epsilon_h = 0$)

$$\text{Hence } 0 = \frac{1}{E} [G_h - \mu (G_h + G_v)]$$

$$\Rightarrow 0 = G_h - \mu (G_h + G_v)$$

$$\Rightarrow G_h = \mu (G_h + G_v)$$

$$\Rightarrow G_h - \mu G_h = \mu G_v$$

$$\Rightarrow G_h (1 - \mu) = \mu G_v$$

$$\Rightarrow \frac{G_h}{G_v} = \left(\frac{\mu}{1 - \mu} \right) = K_0$$

Where K_0 = coefficient of earth pressure at rest

μ = Poisson's ratio

$$\Rightarrow \underline{G_h = K_0 G_v}$$

Designating the lateral earth pressure (G_h) at rest by P_0 and substituting $G_v = \gamma Z$

we have,

$$\boxed{P_0 = K_0 \gamma Z}$$

The pressure distribution diagram is hence triangular with zero intensity at $z=0$ and an intensity of $K_0 \gamma H$ at the base of wall where $z=H$.

The total pressure ' P_0 ' per unit length for the vertical height ' H ' is given by.

$$P_0 = \int_0^H K_0 \gamma z dz = \cancel{K_0 \gamma} \int_0^H z dz$$

$$\boxed{P_0 = \frac{1}{2} K_0 \gamma H^2}$$

Soil type

- 1) loose sand
- 2) Dense sand
- 3) Sand compacted in layers
- 4) Soft clay
- 5) Hard clay

K_0

0.4

0.6

0.8

0.6

0.5

Q.1 A retaining wall 4m high has a smooth, vertical back. The backfill has a horizontal surface in level with the top of the wall. There is uniformly distributed surcharge load of 36 kN/m^2 . The unit weight of backfill is 18 kN/m^2 and angle of shearing resistance is 30° and cohesion is zero. If the water table rises behind the wall to an elevation 1.5m below the top. Determine total active pressure and its point of application. Take submerged weight of sand as 12 kN/m^3 .

Ans let P_1 = Lateral pressure intensity due to surcharge

P_2 = lateral pressure intensity due to dry soil = P_3

P_4 = Lateral pressure intensity due to submerged soil

P_5 = Lateral pressure intensity due to water.

$$P_1 = K_a q = \frac{1}{3} \times 36 = 12 \text{ kN/m}^2$$

$$K_a = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = \frac{1}{3}$$

~~top~~

$$P_2 = K_a \gamma H_1 = \frac{1}{3} \times 18 \times 1.5 = 9 \text{ kN/m}^2$$

$$P_4 = K_a \gamma' H_2 = \frac{1}{3} \times 12 \times 2.5 = 10 \text{ kN/m}^2$$

$$P_5 = \gamma_w H_2 = 9.81 \times 2.5 = 24.53 \text{ kN/m}^2$$

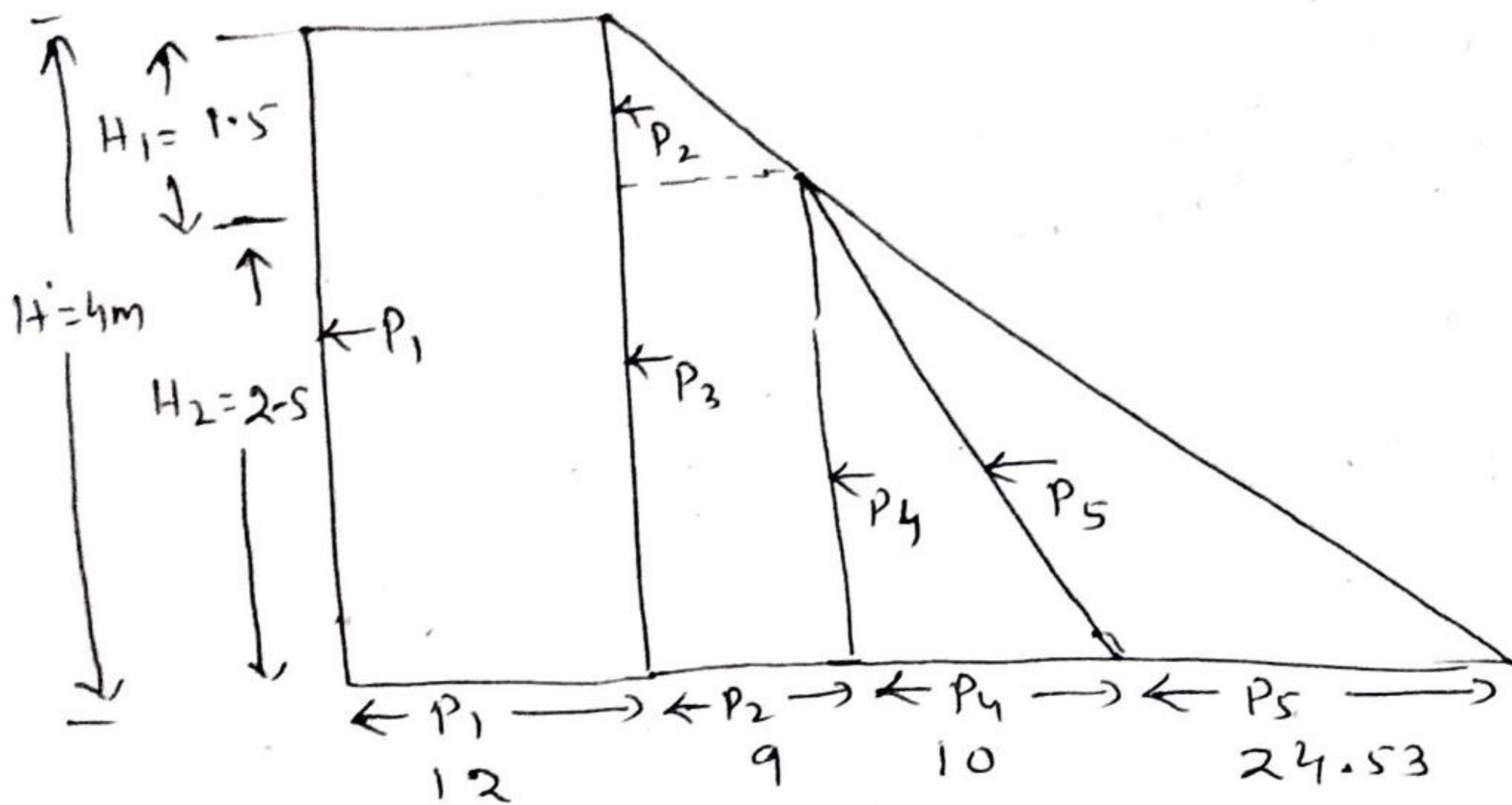


Figure shows the pressure distribution diagram with resultant pressure P_1, P_2, P_3, P_4 and P_5 :

Now total pressure $P_1 = P_1 H = 12 \times 4$
 $= 48 \text{ kN/m}$ acting at $\frac{4}{2} = 2 \text{ m}$ from base.

$P_2 = \frac{1}{2} P_2 H_1 = \frac{1}{2} \times 9 \times 1.5 = 6.75 \text{ kN/m}$
 acting at $2.5 + \frac{1.5}{3} = 3 \text{ m}$ from base

$P_3 = P_2 H_2 = 9 \times 2.5 = 22.5 \text{ kN/m}$ acting at
 $\frac{H_2}{2} = \frac{2.5}{2} = 1.25 \text{ m}$ from base.

$P_4 = \frac{1}{2} P_4 \times H_2 = \frac{1}{2} \times 10 \times 2.5 = 12.5 \text{ kN/m}$
 acting at $\frac{1}{3} H_2 = \frac{1}{3} \times 2.5 = 0.833 \text{ m}$ from base

$P_5 = \frac{1}{2} P_5 H_2 = \frac{1}{2} \times 24.53 \times 2.5 = 30.66 \text{ kN/m}$
 acting at $\frac{1}{3} H_2 = \frac{1}{3} \times 2.5 = 0.833 \text{ m}$ from
 base.

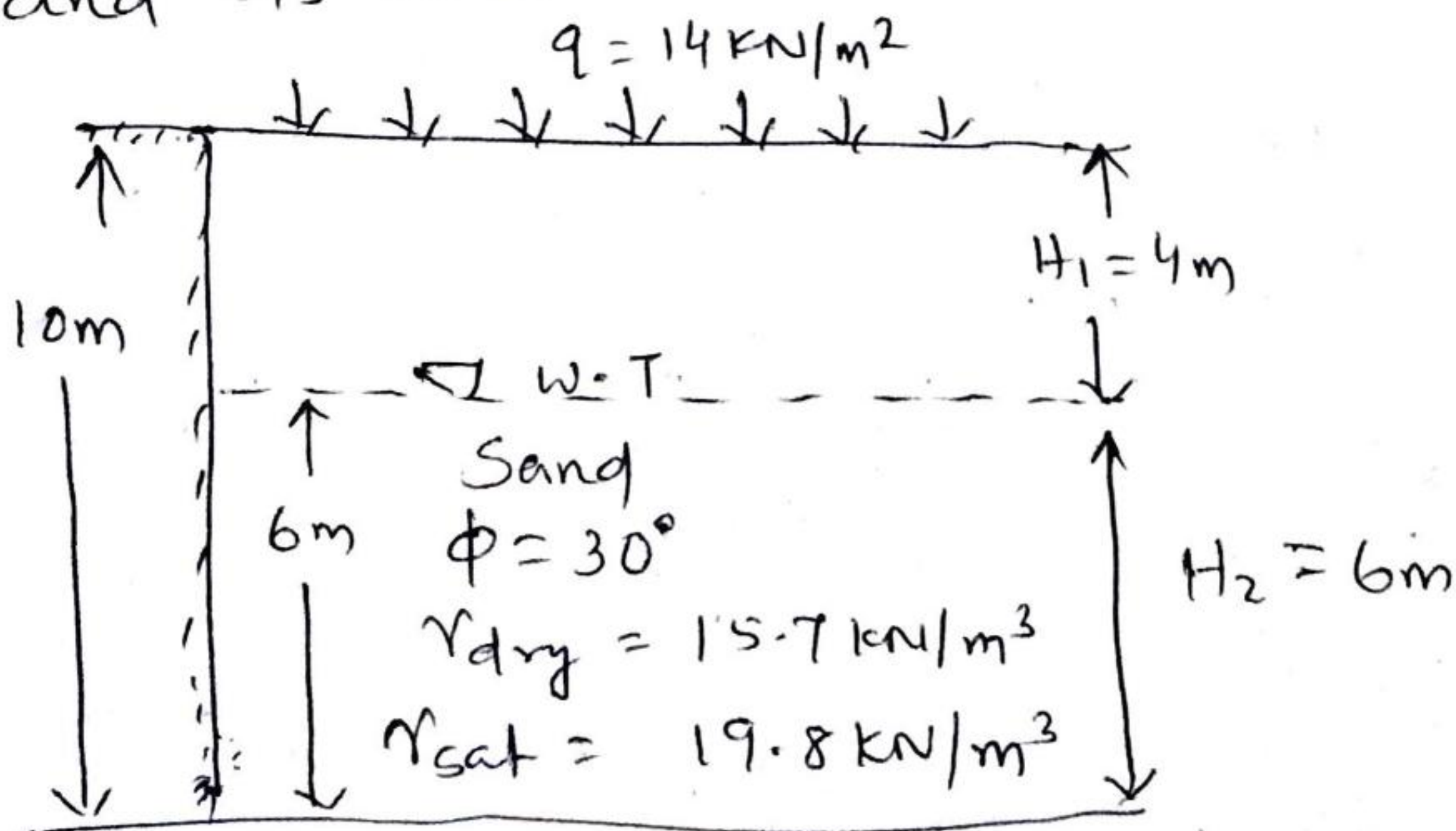
$$\begin{aligned} \text{Total pressure } P &= P_1 + P_2 + P_3 + P_4 + P_5 \\ &= 48 + 6.75 + 22.5 + 12.5 + 30.66 \\ &= 120.41 \text{ kN/m} \end{aligned}$$

The distance \bar{z} of the point of application of P above the base is obtained by taking moments about the base.

$$\bar{z} = \frac{P_1 z_1 + P_2 z_2 + P_3 z_3 + P_4 z_4 + P_5 z_5}{P}$$

$$\begin{aligned} &= \frac{1}{120.41} \left[48 \times 2 + 6.75 \times 3 + 22.5 \times 1.25 + \right. \\ &\quad \left. 12.5 \times 0.833 + 30.66 \times 0.833 \right] \\ &= 1.50 \text{ m} \end{aligned}$$

Q.2 For an earth retaining wall shown in figure, sketch the earth pressure diagram under active state and find the total pressure per unit length of wall and its location.



$$\text{Ans } K_a = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = \frac{1}{3}$$

$$P_1 = K_a q = \frac{1}{3} \times 14 = 4.67 \text{ kN/m}^2$$

$$P_2 = K_a \gamma_d H_1 = \frac{1}{3} \times 15.7 \times 4 = 20.93 \text{ kN/m}^2$$

$$P_3 = P_2 = 20.93 \text{ kN/m}^2$$

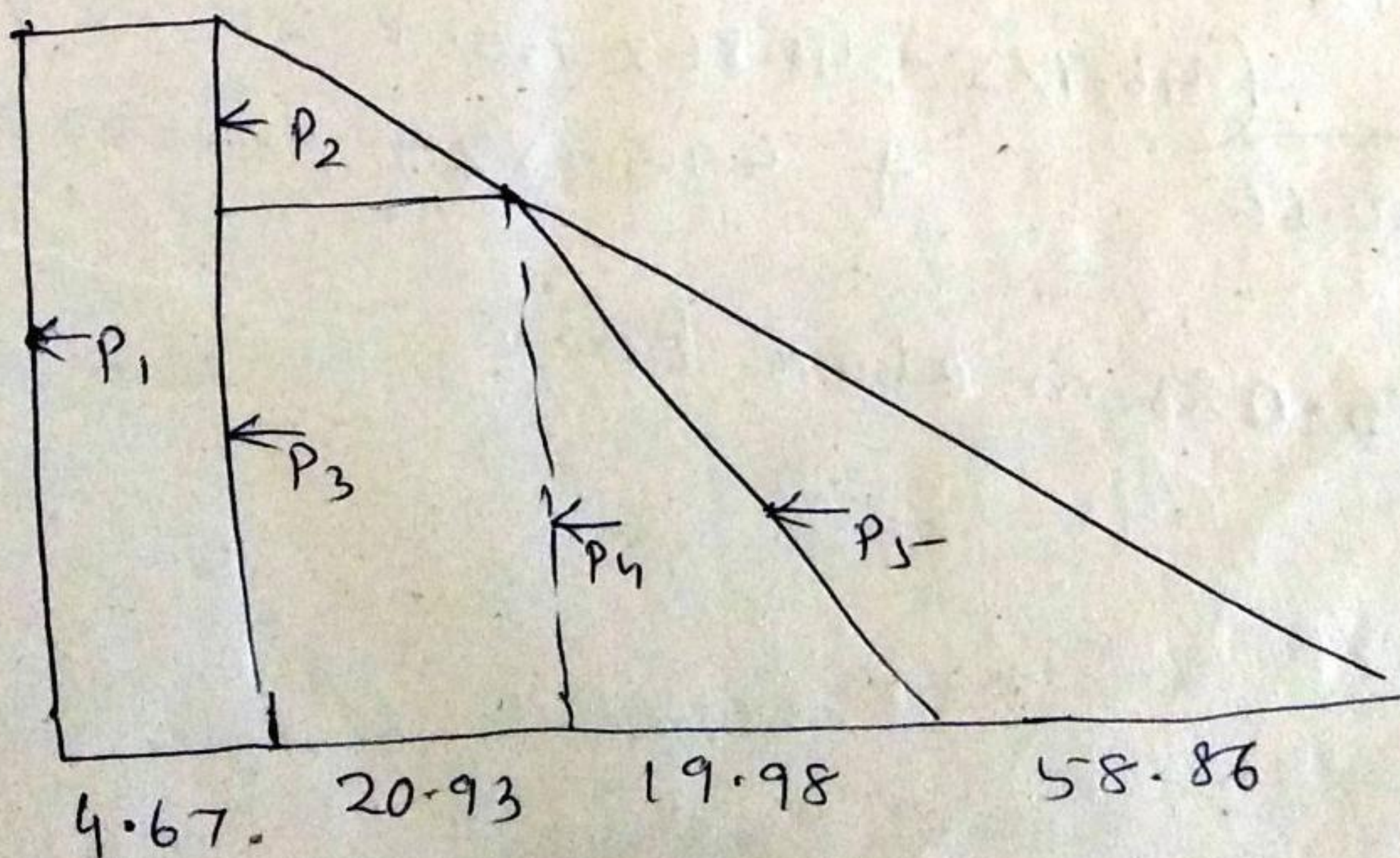
$$P_4 = K_a \gamma' H_2 = \frac{1}{3} (19.8 - 9.81) \times 6 = 19.98 \text{ kN/m}^2$$

$$P_5 = \gamma_w H_2 = 9.81 \times 6 = 58.86 \text{ kN/m}^2$$

Now ~~total~~ total pressure intensity is given by

$$P_1 = P_1 \times H = 4.67 \times 10 = 46.7 \text{ kN/m acting at } z_1 = \frac{10}{2} = 5 \text{ m above base}$$

$$P_2 = \frac{1}{2} P_2 H_1 = \frac{1}{2} \times 20.93 \times 4 = 41.86 \text{ kN/m acting at } z_2 = 6 + \frac{1}{3} \times 4 = 7.33 \text{ m above base}$$



$$P_3 = P_3 H_2 = 20.93 \times 6 = 125.58 \text{ kN/m}$$

acting at $\frac{z_3}{2} = \frac{6}{2} = 3 \text{ m}$ above base

$$P_4 = \frac{1}{2} P_4 \times H_2 = \frac{1}{2} \times 19.98 \times 6 = 59.94 \text{ kN/m}$$

acting at $z_4 = \frac{1}{3} H_2 = \frac{1}{3} \times 6 = 2 \text{ m}$ above base

$$P_5 = \frac{1}{2} P_5 \times H_2 = \frac{1}{2} \times 58.86 \times 6 = 176.58 \text{ kN/m}$$

acting at $z_5 = \frac{1}{3} H_2 = \frac{6}{3} = 2 \text{ m}$ above base.

$$\text{Total } P = P_1 + P_2 + P_3 + P_4 + P_5$$

$$= 46.7 + 41.86 + 125.58 + 59.95 + 176.58$$

$$= 450.66 \text{ kN/m}$$

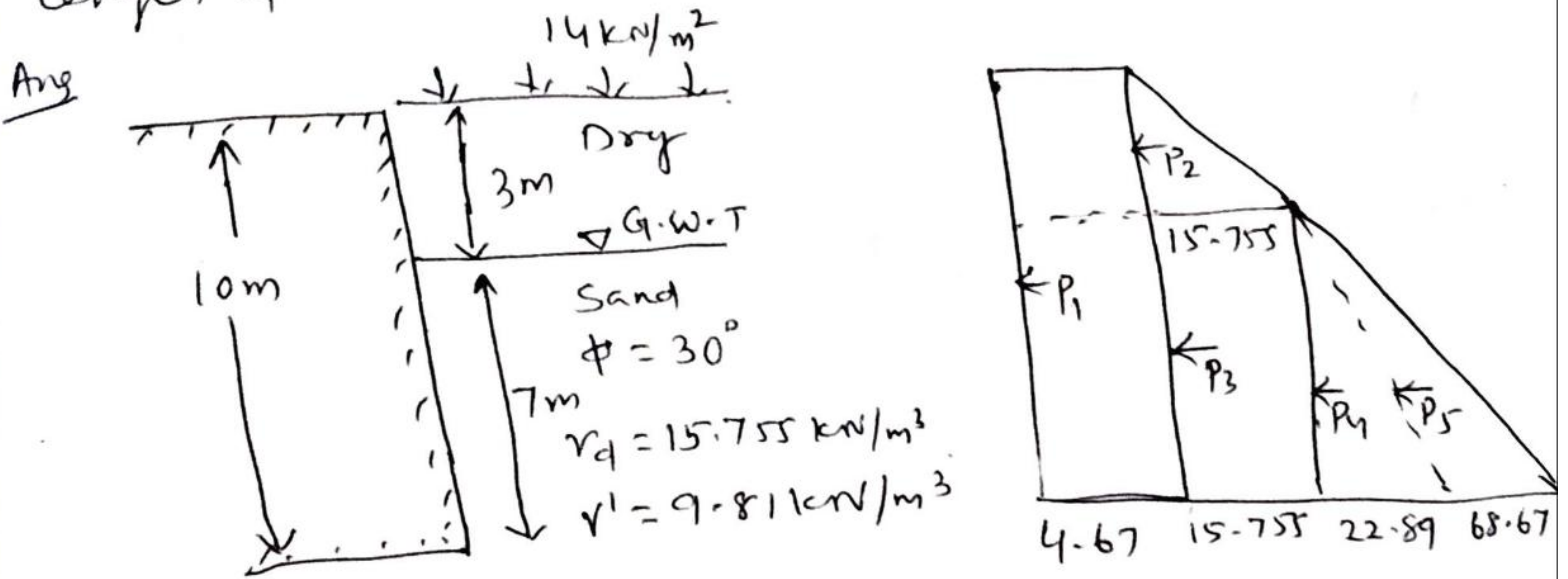
acting at

$$\bar{z} = \frac{1}{P} (P_1 z_1 + P_2 z_2 + P_3 z_3 + P_4 z_4 + P_5 z_5)$$

$$= \frac{1}{450.66} (46.7 \times 5 + 41.86 \times 7.33 + 125.58 \times 3 + 59.94 \times 2 + 176.58 \times 2)$$

$$= 3.085 \text{ m above base}$$

Q-2 For an earth retaining structure shown in figure construct earth pressure diagram for active state and find total thrust per unit length of the wall.



$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 30}{1 + \sin 30} = \frac{1}{3}$$

Lateral earth pressure,

$$P_1 = K_a \times q = \frac{1}{3} \times 14 = 4.67 \text{ kN/m}^2$$

$$P_2 = K_a \gamma_d H_1 = \frac{1}{3} \times 15.755 \times 3 = 15.755 \text{ kN/m}^2 = P_3$$

$$P_4 = K_a \gamma' H_2 = \frac{1}{3} \times 9.81 \times 7 = 22.89 \text{ kN/m}^2$$

$$P_5 = \gamma_w H_2 = 9.81 \times 7 = 68.67 \text{ kN/m}^2$$

Total earth pressure is given by

$$P_1 = 4.67 \times 10 = 46.7 \text{ kN/m}$$

$$P_2 = \frac{1}{2} \times 15.755 \times 3 = 23.632 \text{ kN/m}$$

$$P_3 = \frac{1}{2} \times 15.755 \times 7 = 110.285 \text{ kN/m}$$

$$P_4 = \frac{1}{2} \times 22.89 \times 7 = 80.115 \text{ kN/m}$$

$$P_5 = \frac{1}{2} \times 68.67 \times 7 = 240.345 \text{ kN/m}$$

$$\text{Total } P_a = 46.7 + 23.632 + 110.285 + 80.115 + 240.345$$

$$= 501.08 \text{ kN/m}$$

Bearing Capacity

Footing A footing is a portion of the foundation of a structure that transmits loads directly to the soil.

Foundation A foundation is that part of structure which is in direct contact with and transmits loads to the ground.

Foundation of soil

It is the upper part of ~~soil~~ the earth mass carrying the load of the structure.

Bearing capacity

The supporting power of a soil or rock is referred to as its bearing capacity.

Gross pressure intensity (q)

It is the total pressure at the base of the footing due to the weight of the superstructure, self weight of the footing and the weight of the earth fill.

Net pressure intensity (q_n)

It is defined as the difference in intensities of the gross pressure and the original overburden pressure. If 'D' is the depth of footing, $q_n = q - \bar{\sigma} = q - \gamma D$

γ = average unit weight of soil

Ultimate bearing capacity (q_f)

It is defined as the minimum gross pressure intensity at the base of the foundation at which the soil fails in shear.

Net ultimate bearing capacity (q_{nf})

It is the minimum net pressure intensity causing shear failure of soil.

$$q_f = q_{nf} + \bar{c}$$

Effective surcharge at the base level of foundation (\bar{c})

It is the intensity of vertical pressure at the base level of foundation.

$$\bar{c} = \gamma D$$

Net safe bearing capacity (q_{ns})

It is the ultimate bearing capacity divided by a factor of safety (F)

Mathematically $q_{ns} = \frac{q_{nf}}{F}$

Safe bearing capacity (q_s)

The maximum pressure which the soil can carry safely without risk of failure is called safe bearing capacity.

$$q_s = q_{ns} + \gamma D = \frac{q_{nf}}{F} + \gamma D$$

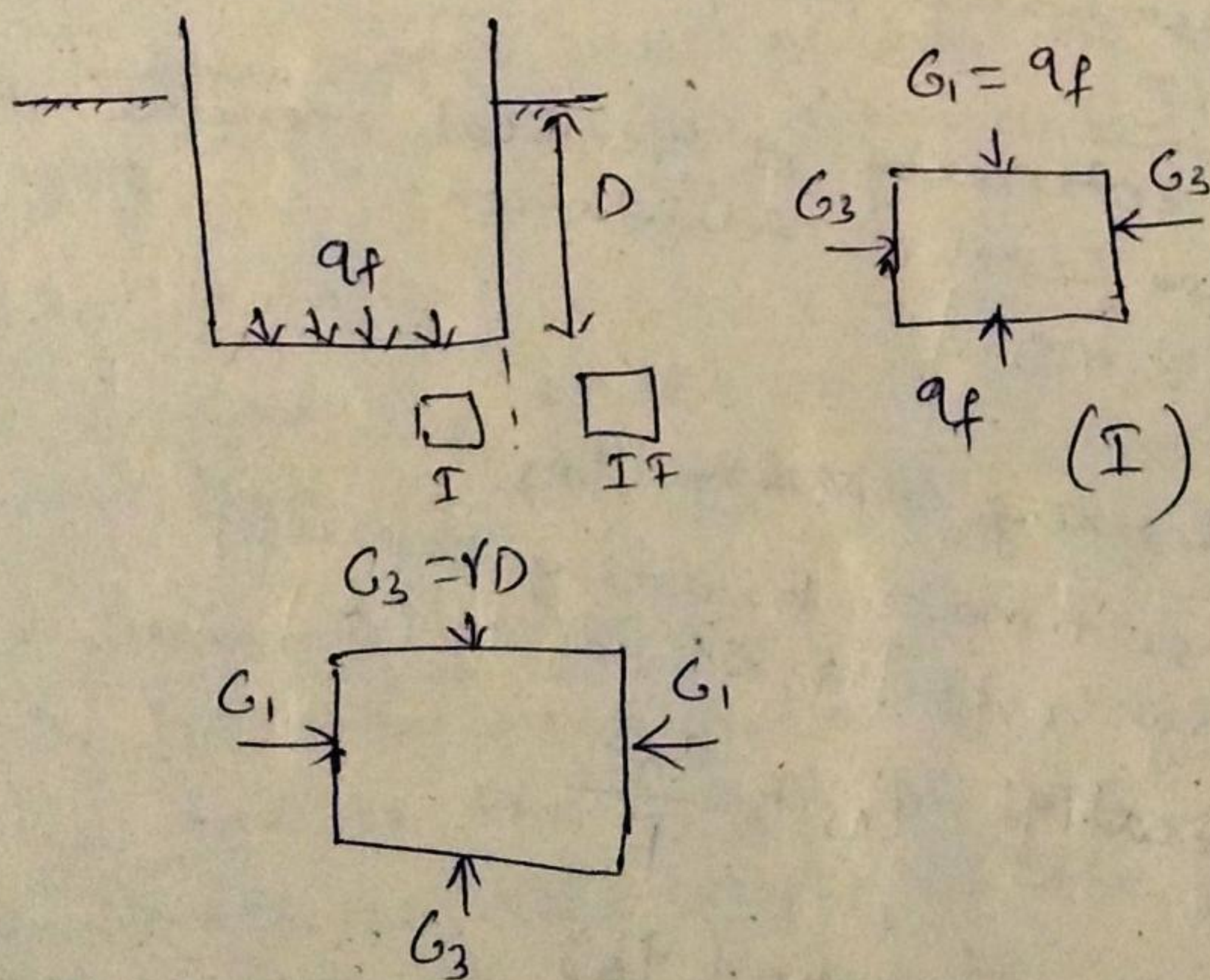
$$q_s = \frac{q_{nf}}{F} + \gamma D \quad (F = \text{factor of safety})$$

Minimum depth of foundation

Rankine's Analysis

When the load on the footing increases and approaches a value q_f a state of plastic equilibrium is reached under the footing.

During the state of shear failure (plastic equilibrium) following principal stress relationship exists,



~~For strength (I)~~ We know that in equilibrium

$$G_1 = G_3 \tan^2 \alpha + 2c \tan \alpha$$

For cohesionless soil $c = 0$

$$G_1 = G_3 \tan^2 \alpha \quad \text{--- (1)}$$

For element (II)

$$G_3 = G_v = \gamma D$$

$$\text{and } G_1 = G_h = \gamma D \tan^2 \alpha \quad \text{--- (2)}$$

But in element (I)

$$G_3 = G_h = G_1 = \gamma D \tan^2 \alpha \quad \text{--- (3)}$$

From equation (1)

$$G_1 = G_3 \tan^2 \alpha$$

Put the value of G_3 from equⁿ (3) we get

$$\textcircled{2} G_1 = (\gamma D \tan^2 \alpha) \tan^2 \alpha$$
$$= \gamma D \tan^4 \alpha$$

$$\text{and } G_1 = q_f$$

$$\text{Hence } q_f = \gamma D \tan^4 \alpha$$

$$q_f = \gamma D (\tan^2 \alpha)^2$$

$$q_f = \gamma D \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right)^2$$

This is the equilibrium of two soil elements one immediately below the foundation (element I) and other just beyond the edge of footing (element II) but adjacent to element (I).

$$\text{Now } D = \frac{q_f}{\gamma} \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)^2$$

Strip footing

A strip footing is a continuous strip of concrete that serves to spread the weight of a load bearing wall across an area of soil.

Strip footings are commonly used as foundations of load bearing walls.

The footing usually has twice the width as the load bearing wall is even wider.

The width as well as type of reinforcement are depending on the bearing capacity of the foundation of soil.

The bearing capacity for strip footing is

$$q_f = \frac{2}{3} c N_c + \gamma D N_q + 0.5 \gamma B N_r$$

Q.1 A strip footing 1m wide at its base is located at a depth of 0.8m below the ground surface. The properties of the foundation soil are, $\gamma = 18 \text{ kN/m}^3$, $c = 30 \text{ kN/m}^2$ and $\phi = 20^\circ$. Determine the safe bearing capacity using a factor of safety of 3. Use Terzaghi's analysis. Assume the soil fails by local shear.

$$\underline{\text{Ans}} \quad q_f = \frac{2}{3} c N_c' + \gamma D N_q' + 0.5 \gamma B N_r'$$

$$\text{For } \phi = 20^\circ$$

$$N_c' = 11.8, \quad N_q' = 3.9$$

$$N_r' = 1.7$$

$$q_f = \frac{2}{3} \times 30 \times 11.8 + 18 \times 0.8 \times 3.9 \\ + 0.5 \times 18 \times 1 \times 1.7$$

$$= 236 + 56.2 + 15.3 = 307.5 \text{ kN/m}^2$$

$$q_{nf} = q_f - \gamma D \\ = 307.5 - 18 \times 0.8 = 293.1 \text{ kN/m}^2$$

$$q_f = \frac{q_{nf}}{F} + \gamma D \\ = \frac{293.1}{3} + 18 \times 0.8 \\ = 112.1 \text{ kN/m}^2$$

Rectangular footing

$$q_f = c N_c \left(1 + 0.3 \frac{B}{L}\right) + \gamma D N_q + 0.4 \gamma B N_r$$

OR

$$q_f = c N_c \left(1 + 0.3 \frac{B}{L}\right) + \gamma D N_q + 0.5 \gamma B N_r \left(1 - 0.2 \frac{B}{L}\right)$$

Q.1 A rectangular footing $2\text{m} \times 3\text{m}$ rests on a cohesion soil with its base at 1.5m below the ground surface.

Calculate the safe bearing capacity using a factor of Safety 3 on
 i) net ultimate bearing capacity and
 ii) Ultimate bearing capacity

Given that $\gamma = 18 \text{ kN/m}^3$ $B = 2\text{m}$
 $c = 10 \text{ kN/m}^2$ $L = 3\text{m}$
 $\phi = 30^\circ$ $D = 1.5\text{m}$

$N_c = 37.2, N_q = 22.5, N_r = 19.7$

$$q_f = cN_c \left(1 + 0.3 \frac{B}{L}\right) + \gamma D N_q + 0.5 \gamma B N_r \left(1 - 0.2 \frac{B}{L}\right)$$

$$= 10 \times 37.2 \left(1 + 0.3 \times \frac{2}{3}\right) + 18 \times 22.5 \times 1.5 + 0.5 \times 18 \times 2 \times 19.7 \left(1 - 0.2 \times \frac{2}{3}\right)$$

$$= 1361.2 \text{ kN/m}^2$$

i) $q_{nf} = q_f - \gamma D = 1361.2 - 18 \times 1.5$
 $= 1334.2 \text{ kN/m}^2$

$q_s = \frac{q_{nf}}{F} + \gamma D = \frac{1334.2}{3} + 18 \times 1.5$
 $= 471.7 \text{ kN/m}^2$

ii) $q_s = \frac{q_f}{3} = \frac{1361.2}{3}$
 $= 453.7 \text{ kN/m}^2$

Q.2 Determine the depth at which a circular footing of 2m diameter be founded to provide a factor of safety of 3, if it has to carry a safe load of 1600 kN. The foundation of soil has $c = 10 \text{ kN/m}^2$, $\phi = 30^\circ$ and unit weight, $= 18 \text{ kN/m}^3$.

Ans

$$d = 2 \text{ m}$$

$$F = 3$$

$$\text{Load} = 1600 \text{ kN}$$

$$c = 10 \text{ kN/m}^2$$

$$\gamma = 18 \text{ kN/m}^3$$

$$\phi = 30^\circ$$

$$N_c = 37.2$$

$$N_q = 22.5$$

$$N_r = 19.7$$

$$q_{nf} = q_f - \gamma D$$

$$= 1.3 c N_c + \gamma D N_q + 0.3 \gamma B N_r - \gamma D$$

$$= 1.3 c N_c + \gamma D (N_q - 1) + 0.3 \gamma B N_r$$

$$= 1.3 \times 10 \times 37.2 + 18 D (22.5 - 1)$$

$$+ 0.3 \times 18 \times 2 \times 19.7$$

$$= 696.36 + 387 D$$

$$q_s = \frac{q_{nf}}{F} + \gamma D = \frac{696.36 + 387 D}{3} + 18 D$$

$$= \frac{697.36 + 387 D + 54 D}{3} = \frac{697.36}{3} + \frac{441 D}{3}$$

$$= 232.12 + 147 D$$

$$\text{Load} = q_s \times \text{area} = (232.12 + 147 D) \times \frac{\pi}{4} \times (2)^2$$

$$\Rightarrow 1600 = (232.12 + 147 D) \times \frac{\pi}{4} \times (2)^2$$

$$\Rightarrow 232.12 + 147 D = \frac{1600}{\pi/4 \times (2)^2}$$

$$\text{Solving } D = 1.89 \text{ m}$$

$$= 1.9 \text{ m}$$

Foundation

Foundations may be broadly classified under two categories.

- 1) shallow foundation
- 2) Deep foundation

A foundation is said to be shallow if its depth is equal to or less than its width.

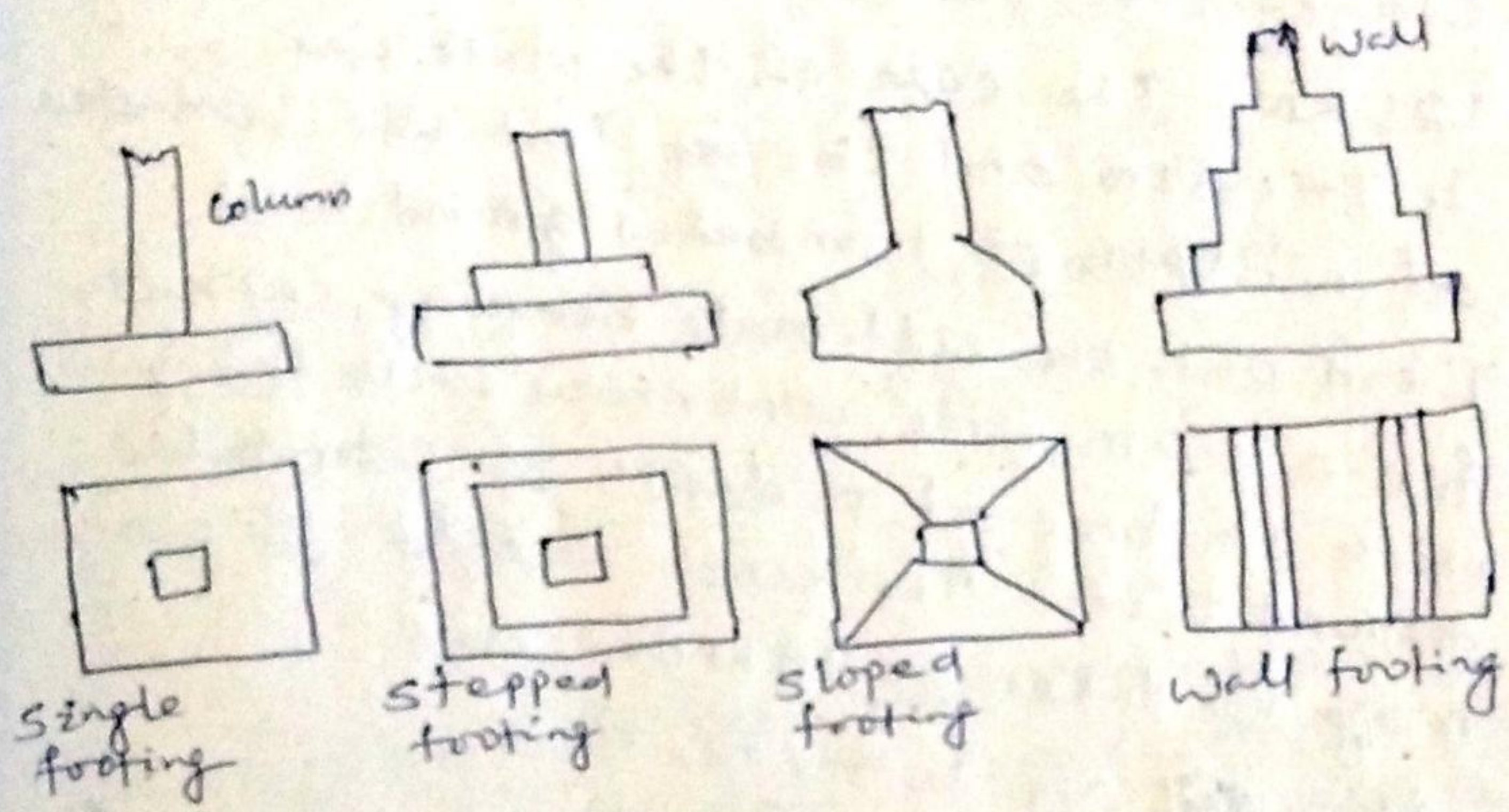
A foundation is said to be deep if its depth is equal to or greater than the width.

Other common forms of deep foundations are pier foundation, pile foundation, and well foundation.

The shallow foundations are of the following types.

- i) spread footing
- ii) strap footing
- iii) combined footing
- iv) mat or raft footing.

Spread footing



A spread footing ~~or foot~~ is a type of shallow foundation used to transmit the load of an isolated column or that of a wall to the subsoil. This is most common type of ~~fund~~ foundation. The base of the column or wall is enlarged or spread to provide individual support for load.

Terzaghi's theory of one-dimensional consolidation

In the development of mathematical ~~statement~~ statement of the consolidation process, the following simplifying assumptions are made:

- i) The soil is homogeneous and fully saturated.
- ii) Soil particles and water are incompressible.
- iii) The deformation of the soil is due entirely to change in volume.
- iv) Darcy's Law for the velocity of flow of water through soil is perfectly valid.
- v) Coefficient of permeability is constant.
- vi) Load is applied in one direction only.
- vii) Excess pore water drains out only in vertical direction.
- viii) The boundary is a free surface.

Coefficient of consolidation is written as

$$c_v = \frac{d^2}{T_v}$$

Where c_v = coefficient of consolidation
 t = time required to attain a certain degree of consolidation is directly proportional to the square of drainage path.
Inversely proportional to coefficient of consolidation.

C_v is assumed as constant quantity and in some cases it is variable quantity. C_v increases with increasing magnitude of consolidating pressure.

$T_v =$ Time factor

Q.1 An undisturbed sample of clay 24mm

thick consolidated 50% in 20 minutes, when tested in the laboratory with drainage allowed at top and bottom.

The clay layer from which the sample was obtained is 4m thick in the field.

How much will it take to consolidate 50% with double drainage.

If the clay stratum has only single drainage, calculate the time to consolidate 50%. Assume uniform distribution of consolidation pressure.

Ans For the same degree of consolidation

T_v is same.

Hence $t \propto \frac{d^2}{C_v}$

Also since both soils are the same

$t \propto d^2$

(a) For the same case of double drainage

$$\left(\frac{t_2}{t_1}\right) = \left(\frac{d_2}{d_1}\right)^2$$

where d_2 = drainage path in the field
 $= \frac{4}{2} \text{ m} = 200 \text{ cm}$

d_1 = drainage path in laboratory specimen
 $= \frac{2.4}{2} = 1.2 \text{ cm}$

t_1 = time for 50% consolidation in the laboratory = 20 min

$$t_2 = t_1 \left(\frac{d_2}{d_1}\right)^2 = 20 \left(\frac{200}{1.2}\right)^2 \text{ minutes}$$
$$= 386 \text{ days}$$

b) For the case of single drainage
 $d_2 = 4 \text{ m} = 400 \text{ cm}$

$$t_2 = 20 \left(\frac{400}{1.2}\right)^2 \text{ minutes}$$
$$= 1544 \text{ days.}$$